Test answers - Trigonometry in right-angled triangles

Year: Year 9 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. Label the sides from angle A: the 5 cm side is opposite, the 12 cm side is adjacent and the 13 cm side is the hypotenuse. Then $\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13}$ (1 mark), $\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}$ (1 mark) and $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{5}{12}$ (1 mark).

b. Use the inverse tangent: $A = \tan^{-1}\left(\frac{5}{12}\right) = 22.61\ldots$, so 22.6°. (1 mark)

The inverse sine or cosine give the same angle: $\sin^{-1}\left(\frac{5}{13}\right) = 22.6^\circ$. The other acute angle is $90 - 22.6 = 67.4^\circ$. Make sure the calculator is in degree mode.

Answer to Question 2 (4 marks)

a. The hypotenuse is known and the opposite side is wanted, so use sine: $\sin 35^\circ = \frac{\text{opp}}{12}$, which gives opp $= 12 \sin 35^\circ$ (1 mark) $= 6.882\ldots$, so **6.88 cm** (1 mark).

b. The hypotenuse is known and the adjacent side is wanted, so use cosine: adj $= 12 \cos 35^\circ$ (1 mark) $= 9.829\ldots$, so **9.83 cm** (1 mark).

Check with Pythagoras: $6.88^2 + 9.83^2 = 47.3 + 96.6 = 143.9$, very close to $12^2 = 144$; the small difference comes from rounding. SOH CAH TOA helps choose the ratio: write down which two sides are involved, then pick the ratio that uses them.

Answer to Question 3 (4 marks)

a. The opposite and adjacent sides are known, so use tangent: $\tan\theta = \frac{7}{9}$ (1 mark). Then $\theta = \tan^{-1}\left(\frac{7}{9}\right) = 37.87\ldots$, which is 37.9° (1 mark).

b. The adjacent side and the hypotenuse are known, so use cosine: $\cos\theta = \frac{11}{15}$ (1 mark). Then $\theta = \cos^{-1}\left(\frac{11}{15}\right) = 42.83\ldots$, which is 42.8° (1 mark).

Sense check for (a): the opposite side is shorter than the adjacent side, so θ must be

less than 45°, and 37.9° fits. In (b) the ratio 11/15 is about 0.73; since $\cos 45^\circ \approx 0.71$, the angle should be close to 45°, which it is.

Answer to Question 4 (4 marks)

a. The ladder is the hypotenuse, 6 m, and the height up the wall is opposite the 70° angle at the ground. Use sine: height $= 6 \sin 70^\circ$ (1 mark) $= 5.638\ldots$, so **5.64 m** (1 mark).
b. The distance from the wall is adjacent to the 70° angle. Use cosine: distance $= 6 \cos 70^\circ$ (1 mark) $= 2.052\ldots$, so **2.05 m** (1 mark).

Check with Pythagoras: $5.64^2 + 2.05^2 = 31.8 + 4.2 = 36.0 = 6^2$. A steep angle like 70° should give a tall reach and a short distance from the wall, which agrees with these answers. Safety guides recommend an angle of about 75° for ladders.

Answer to Question 5 (4 marks)

a. From Sam's eyes, the horizontal distance of 45 m is adjacent to the 28° angle and the height above eye level is opposite. Use tangent: height $= 45 \tan 28^\circ$ (1 mark) $= 23.93\ldots$, so **23.9 m** (1 mark).
b. Sam measured from eye level, so add his eye height: $23.9 + 1.6 =$**25.5 m** above the ground. (1 mark)
c. Now the opposite side is 30 m and the adjacent side is 45 m:
$\tan^{-1}\left(\frac{30}{45}\right) = 33.69\ldots$, so 33.7°. (1 mark)

Forgetting the 1.6 m in (b) is the usual error. A taller tower gives a larger angle of elevation from the same distance.