Worksheet - Trigonometry in right-angled triangles

Year: Year 9 · 20

Show your working.

Question 1

Question 1 – Naming the sides *(Section A · Skills practice)*

In a right-angled triangle the hypotenuse is opposite the right angle. The opposite side is across from the marked angle, and the adjacent side is next to it (not the hypotenuse).

a. Triangle ABC has a right angle at B. For angle A, name the hypotenuse, the opposite side and the adjacent side.
b. For angle C in the same triangle, name the opposite and adjacent sides.
c. Triangle PQR has a right angle at Q. For angle P, which side is opposite?
d. Which side is the hypotenuse of triangle PQR?

Question 2

Question 2 – Writing the ratios *(Section A · Skills practice)*

SOH CAH TOA: $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$. A right-angled triangle has sides 3, 4 and 5. The angle θ is opposite the side 3.

a. Write $\sin\theta$, $\cos\theta$ and $\tan\theta$ as fractions.
b. Write the same three ratios for the other acute angle ϕ.
c. What do you notice about $\sin\theta$ and $\cos\phi$?
d. A triangle has sides 5, 12, 13. Write $\sin$, $\cos$ and $\tan$ of the angle opposite 5.

Question 3

Question 3 – Using the calculator *(Section A · Skills practice)*

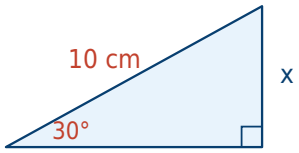
Make sure your calculator is in degree mode. Give each answer to 3 decimal places.

a. $\sin 30^\circ$ and $\cos 60^\circ$
b. $\tan 45^\circ$
c. $\sin 40^\circ$ and $\cos 25^\circ$
d. $\tan 70^\circ$

Question 4

Question 4 – Finding a missing side with sine *(Section A · Skills practice)*

The hypotenuse and an angle are known and you need the opposite side, so use $\text{opp} = \text{hyp} \times \sin\theta$. Give answers to 2 decimal places.



a. Hypotenuse 10 cm, angle 30°.
b. Hypotenuse 8 cm, angle 40°.
c. Hypotenuse 15 cm, angle 55°.
d. Hypotenuse 20 cm, angle 25°.

Question 5

Question 5 – Choosing sine, cosine or tangent *(Section A · Skills practice)*

Decide which ratio to use, write the equation and find the length x. Give answers to 2 decimal places.

a. Adjacent 6 cm, angle 35°. Find the opposite side x.
b. Adjacent 8 cm, angle 50°. Find the hypotenuse x.
c. Opposite 7 cm, angle 40°. Find the adjacent side x.
d. Adjacent x is needed from hypotenuse 12 cm and angle 62°.

Question 6

Question 6 – Finding an angle *(Section A · Skills practice)*

Use the inverse functions $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ to find the angle. Give answers to 1 decimal place.

a. $\sin\theta = 0.5$
b. $\cos\theta = 0.8$
c. $\tan\theta = 1.5$
d. A right-angled triangle has opposite 6 cm and hypotenuse 10 cm. Find θ.

Question 7

Question 7 – The ladder angle *(Section B · Real-life practice)*

A 5 m ladder leans against a vertical wall. The foot of the ladder is 1.5 m from the wall.

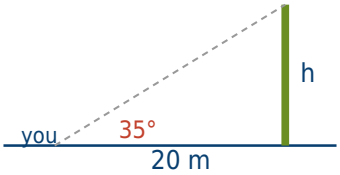
a. Find the angle between the ladder and the ground, to 1 decimal place.

b. Find how high up the wall the ladder reaches, using sine.
c. Check your answer to (b) using Pythagoras' theorem.
d. Safety guidance says the angle should be between 70° and 80°. Is the ladder placed safely?

Question 8

Question 8 – How tall is the tree? *(Section B · Real-life practice)*

Layla stands 20 m from the base of a tree. She measures the angle of elevation of the top of the tree as 35°. Her eyes are 1.6 m above the ground.



a. Find the height h of the top of the tree above her eye level.
b. Find the height of the tree above the ground.
c. She moves back until the angle is 25°. How far from the tree is she now?
d. Why is it important to add the height of her eyes?

Question 9

Question 9 – The wheelchair ramp *(Section B · Real-life practice)*

A ramp is 4 m long and rises by 0.5 m.

a. Find the angle of the ramp to the horizontal.
b. Find the horizontal distance covered by the ramp.
c. The rule says the angle must not be more than 5°. Is it allowed?
d. What height would a ramp of the same length have at 5°?

Question 10

Question 10 – Flying a kite *(Section B · Real-life practice)*

Rhys flies a kite on a 60 m string. The string makes an angle of 50° with the horizontal ground. (Ignore the sag in the string.)

a. Find the height of the kite above the ground, to the nearest metre.
b. Find how far along the ground the kite is from Rhys.
c. The angle later changes to 65° with the string length the same. Find the new height.
d. Which is higher, and by how much?

Question 11

Question 11 – The playground slide *(Section B · Real-life practice)*

A playground slide is 3 m long and makes an angle of 35° with the ground.

a. Find the height of the top of the slide.
b. Find the horizontal distance from the foot of the slide to the point below the top.
c. Check using Pythagoras.
d. A second slide is 3 m long but only 1.2 m high. Find its angle.

Question 12

Question 12 – The plane descends *(Section B · Real-life practice)*

A plane descends at an angle of 3° to the horizontal while it travels a horizontal distance of 30 km towards the airport.

a. Find the height lost, to 2 decimal places.
b. Find the distance flown through the air.
c. The plane starts at 10 km high. How far must it travel horizontally to reach the ground at 3°?
d. Why are descents shallow in practice?

Question 13

Question 13 – The shadow and the sun *(Section B · Real-life practice)*

Ben is 1.8 m tall and his shadow on level ground is 2.5 m long.

a. Find the angle of elevation of the sun, to 1 decimal place.
b. The shadow of a tower at the same moment is 40 m. Find the height of the tower.
c. Later the elevation is 50°. How long is Ben's shadow?
d. Is the shadow longer or shorter when the sun is higher?

Question 14

Question 14 – The cable car *(Section B · Real-life practice)*

A cable car climbs at 28° to the horizontal up to a mountain station 600 m above the start.

a. Find the length of the cable.
b. Find the horizontal distance between the stations.
c. The cable car travels at 4 m/s. How long is the journey, in minutes and seconds?
d. A steeper cable at 35° to the same height is built. Find its length.

Question 15

Question 15 – Triangle fact table *(Section C · Problem solving)*

The table shows three right-angled triangles. The hypotenuse is h, the opposite side is o and the adjacent side is a for the angle θ. All lengths are in centimetres.

Triangle	θ	h	o	a
1	30°	12	?	?
2	40°	?	9	?
3	?	10	6	8

a. Complete the table. Give answers to 1 decimal place.
b. Check triangle 1 using Pythagoras.
c. Show that triangle 3 is a 3, 4, 5 triangle enlarged. By what scale factor?
d. Find the other acute angle in triangle 3.

Question 16

Question 16 – The isosceles triangle *(Section C · Problem solving)*

An isosceles triangle has two equal sides of 10 cm and an angle of 40° between them. The line from the apex to the midpoint of the base splits it into two right-angled triangles.

a. Find the angle at the apex of each right-angled triangle.
b. Find the half-base and the height.
c. Find the base of the isosceles triangle.
d. Find the area of the isosceles triangle.
e. Find the perimeter.

Question 17

Question 17 – The tower and flagpole *(Section C · Problem solving)*

Anna stands 30 m from the foot of a building. She measures the angle of elevation to the top of the building as 52° and to the top of a flagpole on the roof as 57°.

a. Find the height of the building.
b. Find the height of the top of the flagpole above the ground.
c. Find the height of the flagpole.
d. The flagpole is meant to be 8 m tall. Is the error in the measurement reasonable?
e. Anna's eyes are 1.5 m above the ground. Give the true height of the building.

Question 18

Question 18 – The lighthouse *(Section C · Problem solving)*

From the top of a lighthouse 40 m above sea level, the angle of depression of a boat is 12°.

a. Explain why the angle between the line of sight and the vertical is not the angle of depression.
b. Find the horizontal distance from the lighthouse to the boat.
c. A second boat in the same direction has an angle of depression of 20°. Find its distance.
d. Find the distance between the two boats.
e. Which boat is closer to the lighthouse?

Question 19

Question 19 – The sloping roof *(Section C · Problem solving)*

The end wall of a house is an isosceles triangle. The span is 8 m and the roof rises 2.4 m above the horizontal at the centre.

a. Find the roof angle to the horizontal.
b. Find the rafter length (the sloping side).
c. Tiles cost £14 per square metre. The roof is two rectangles each as long as a rafter and 10 m deep. Find the area and the cost.
d. A different roof has an angle of 40° and the same span. Find its rise.

Question 20

Question 20 – Challenge: exact values and an identity *(Section D · Challenge)*

In a right-angled isosceles triangle with two equal sides of length 1, the hypotenuse is $\sqrt{2}$. In an equilateral triangle of side 2 split in half, the height is $\sqrt{3}$.

a. Write the exact value of $\sin 45^\circ$ and check it with a calculator.
b. Use the half-equilateral triangle to find the exact values of $\sin 30^\circ$ and $\cos 30^\circ$.
c. Show that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ for the 5, 12, 13 triangle with θ opposite 5.
d. Show that $\sin^2\theta + \cos^2\theta = 1$ for the same triangle.
e. Use the identity to find $\cos\theta$ if $\sin\theta = 0.6$ and θ is acute.