Answers - Trigonometry in right-angled triangles

Year: Year 9

Answer to Question 1

a. Hypotenuse AC; opposite BC; adjacent AB. The hypotenuse is always opposite the right angle at B; the opposite side does not touch angle A.
b. For angle C: opposite AB; adjacent BC. Changing the angle changes which side is opposite and which is adjacent, but the hypotenuse stays the same.
c. Side QR (the side not touching P). The opposite side is the one across the triangle from P, not touching it.
d. PR, opposite the right angle at Q. The hypotenuse is the longest side and is always opposite the right angle.

Answer to Question 2

a. $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$. Side 3 is opposite θ, side 4 is adjacent and side 5 is the hypotenuse.
b. Opposite 4, adjacent 3: $\sin\phi = \frac{4}{5}$, $\cos\phi = \frac{3}{5}$, $\tan\phi = \frac{4}{3}$. For the other angle the opposite and adjacent sides swap, but the hypotenuse stays 5.
c. $\sin\theta = \cos\phi = \frac{3}{5}$: the sine of one angle equals the cosine of the other. The two acute angles add up to 90°, so the sine of one is always the cosine of the other.
d. $\sin = \frac{5}{13}$, $\cos = \frac{12}{13}$, $\tan = \frac{5}{12}$. Side 5 is opposite, side 12 is adjacent and side 13 is the hypotenuse.

Answer to Question 3

a. $\sin 30^\circ =$ **0.500** and $\cos 60^\circ =$ **0.500** These are exact values: $\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$, because 30° and 60° add up to 90°.
b. **1.000** In a right-angled isosceles triangle the opposite and adjacent sides are equal, so $\tan 45^\circ = 1$.
c. $\sin 40^\circ =$ **0.643**, $\cos 25^\circ =$ **0.906** Read four decimal places from the calculator and round to three.
d. **2.747** The tangent of an angle can be bigger than 1, because the opposite side can be longer than the adjacent side.

Answer to Question 4

a. $10 \times \sin 30^\circ =$ **5 cm** $\sin 30^\circ = 0.5$, so the opposite side is half the hypotenuse.
b. $8 \times \sin 40^\circ =$ **5.14 cm** $\sin 40^\circ = 0.6427\ldots$, so the opposite side is $5.142\ldots$ cm.
c. $15 \times \sin 55^\circ =$ **12.29 cm** $\sin 55^\circ = 0.8191\ldots$, so the opposite side is $12.287\ldots$ cm.
d. $20 \times \sin 25^\circ =$ **8.45 cm** $\sin 25^\circ = 0.4226\ldots$, so the opposite side is $8.452\ldots$ cm.

Answer to Question 5

a. Use tan: $x = 6 \tan 35^\circ =$ **4.20 cm** The adjacent side is known and the opposite side is wanted, so TOA links them.
b. Use cos: $\cos 50^\circ = \dfrac{8}{x}$, so $x = 8 \div \cos 50^\circ =$ **12.45 cm** The adjacent side is known and the hypotenuse is wanted, so use CAH and divide because x is on the bottom.
c. Use tan: $\tan 40^\circ = \dfrac{7}{x}$, so $x = 7 \div \tan 40^\circ =$ **8.34 cm** The opposite side is known and the adjacent side is wanted; x is on the bottom, so divide.
d. Use cos: $x = 12 \cos 62^\circ =$ **5.63 cm** The hypotenuse is known and the adjacent side is wanted, so CAH gives $x = 12 \cos 62^\circ$.

Answer to Question 6

a. $\theta = \sin^{-1} 0.5 = 30.0^\circ$ Press shift and sin, then enter 0.5.
b. $\theta = \cos^{-1} 0.8 = 36.9^\circ$ Use the inverse cosine of 0.8 and round to 1 decimal place.
c. $\theta = \tan^{-1} 1.5 = 56.3^\circ$ Use the inverse tangent of 1.5; the angle is more than 45° because the tangent is more than 1.
d. $\sin\theta = \dfrac{6}{10} = 0.6$, so $\theta = 36.9^\circ$ The opposite side and hypotenuse are known, so use sine, then the inverse function.

Answer to Question 7

a. $\cos\theta = \dfrac{1.5}{5} = 0.3$, so $\theta = 72.5^\circ$ The adjacent side (1.5 m from the wall) and the hypotenuse (5 m) are known, so use cosine.
b. $5 \sin 72.5^\circ =$ **4.77 m** The height is opposite the angle, and the hypotenuse is 5 m.
c. $\sqrt{25 - 2.25} = \sqrt{22.75} = 4.77$ m. They agree. Pythagoras gives the same height, which confirms the answer.
d. 72.5° is between 70° and 80°, so **yes**. The angle of 72.5° lies inside the safe range.

Answer to Question 8

a. $h = 20 \tan 35^\circ =$ **14.0 m** The 20 m distance is adjacent to the 35° angle and h is opposite, so use tangent.
b. $14.0 + 1.6 =$ **15.6 m** Add Layla's eye height to the height above her eyes.
c. $\tan 25^\circ = \dfrac{14.0}{d}$, so $d = 14.0 \div \tan 25^\circ =$ **30.0 m**. The height above eye level stays the same, but the angle is smaller, so she must be further away.
d. The angle is measured from her eyes, not from the ground. The triangle starts at her eyes, so the part of the tree below her eye level is not included in h.

Answer to Question 9

a. $\sin\theta = \dfrac{0.5}{4} = 0.125$, so $\theta = 7.2^\circ$ The rise is opposite the angle and the ramp is the hypotenuse, so use sine.
b. $4 \cos 7.18^\circ =$ **3.97 m** The horizontal distance is adjacent to the angle, so use cosine with the hypotenuse.
c. $7.2^\circ > 5^\circ$, so it is **not allowed**. Compare the angle with the maximum allowed.
d. $4 \sin 5^\circ =$ **0.35 m** Use sine again with the same ramp length and an angle of 5°.

Answer to Question 10

a. $60 \sin 50^\circ = 45.96 =$ **46 m** The height is opposite the angle and the string is the hypotenuse.
b. $60 \cos 50^\circ =$ **38.6 m** The distance along the ground is adjacent to the angle.
c. $60 \sin 65^\circ =$ **54.4 m** Use the same method with the new angle of 65°.
d. The second, by $54.4 - 46.0 =$ **8.4 m**. A larger angle with the same string makes the kite higher.

Answer to Question 11

a. $3 \sin 35^\circ =$ **1.72 m** The height is opposite the 35° angle and the slide is the hypotenuse.
b. $3 \cos 35^\circ =$ **2.46 m** The horizontal distance is adjacent to the angle, so use cosine.
c. $1.72^2 + 2.46^2 = 2.96 + 6.05 = 9.01 \approx 3^2$. The two shorter sides squared add up to about $3^2 = 9$.
d. $\sin\theta = \dfrac{1.2}{3} = 0.4$, so $\theta = 23.6^\circ$. Opposite and hypotenuse are known, so use the inverse sine.

Answer to Question 12

a. $30 \tan 3^\circ =$ **1.57 km** The height lost is opposite the angle and the 30 km is adjacent, so use tangent.
b. $30 \div \cos 3^\circ =$ **30.04 km** The path through the air is the hypotenuse; use cosine with x on the bottom.
c. $10 \div \tan 3^\circ =$ **190.8 km** Now the opposite side is 10 km and the adjacent side is unknown, so divide by $\tan 3^\circ$.
d. A shallow angle is comfortable for passengers and keeps the speed controllable. A small angle means the height changes slowly over a long distance.

Answer to Question 13

a. $\tan\theta = \dfrac{1.8}{2.5} = 0.72$, so $\theta = 35.8^\circ$ Ben's height is opposite the angle and his shadow is adjacent, so use the inverse tangent.
b. $40 \times 0.72 =$ **28.8 m** The sun's angle is the same, so the tower's height is 0.72 times its shadow.
c. $\tan 50^\circ = \dfrac{1.8}{s}$, so $s = 1.8 \div \tan 50^\circ =$ **1.51 m** The shadow is adjacent to the angle, so divide the height by $\tan 50^\circ$.
d. **Shorter** when the sun is higher. A higher sun gives a larger angle of elevation, so the adjacent side (the shadow) is shorter.

Answer to Question 14

a. $\sin 28^\circ = \dfrac{600}{L}$, so $L = 600 \div \sin 28^\circ =$ **1 278 m** The height of 600 m is opposite the angle and the cable is the hypotenuse; divide by $\sin 28^\circ$.
b. $600 \div \tan 28^\circ =$ **1 128 m** The horizontal distance is adjacent to the angle, so divide the height by $\tan 28^\circ$.
c. $1278 \div 4 = 319.5$ s = **5 minutes 20 seconds** Time = distance ÷ speed; 319.5 s is 5 minutes and 19.5 seconds, about 5 minutes 20 seconds.
d. $600 \div \sin 35^\circ =$ **1 046 m** A steeper cable reaches the same height in a shorter distance.

Answer to Question 15

a. Triangle 1: $o = 12 \sin 30^\circ = 6.0$, $a = 12 \cos 30^\circ = 10.4$. Triangle 2: $h = 9 \div \sin 40^\circ = 14.0$, $a = 9 \div \tan 40^\circ = 10.7$. Triangle 3: $\theta = \sin^{-1} 0.6 = 36.9^\circ$. Choose the ratio that links the known side with the wanted side in each triangle.
b. $6^2 + 10.39^2 = 36 + 108 = 144 = 12^2$. The squares of the two shorter sides add up to the square of the hypotenuse.

c. Sides $6, 8, 10$ are twice $3, 4, 5$: scale factor **2**. Divide each side by 2 to get 3, 4 and 5.
d. $90 - 36.9 = 53.1^\circ$ The two acute angles of a right-angled triangle add up to 90°.

Answer to Question 16

a. $40 \div 2 = 20^\circ$ The line from the apex is a line of symmetry, so it halves the 40° angle.
b. Half-base $= 10 \sin 20^\circ = 3.42$ cm. Height $= 10 \cos 20^\circ = 9.40$ cm. In each right-angled triangle the hypotenuse is 10 cm; the half-base is opposite 20° and the height is adjacent.
c. Base $= 2 \times 3.42 =$ **6.84 cm** The base is two half-bases.
d. Area $= \frac{1}{2} \times 6.84 \times 9.40 =$ **32.1 cm²** Area of a triangle = $\frac{1}{2}$ × base × height.
e. $10 + 10 + 6.84 =$ **26.84 cm** Add the two equal sides and the base.

Answer to Question 17

a. $30 \tan 52^\circ =$ **38.4 m** The 30 m is adjacent to the angle and the height is opposite, so use tangent.
b. $30 \tan 57^\circ =$ **46.2 m** Use the same method with the larger angle of 57°.
c. $46.2 - 38.4 =$ **7.8 m** The flagpole is the difference between the two heights.
d. 7.8 m is close to 8 m, so the measurement is **reasonable**. The answer is within 0.2 m of the expected value, which is a small error.
e. $38.4 + 1.5 =$ **39.9 m** The angles were measured from eye level, so add her eye height.

Answer to Question 18

a. The angle of depression is measured from the horizontal, down to the line of sight; by alternate angles it equals the angle of elevation from the boat. The angle of depression is measured down from the horizontal, not from the vertical.
b. $40 \div \tan 12^\circ =$ **188.2 m** The 40 m height is opposite the angle at the boat, and the distance is adjacent, so divide by $\tan 12^\circ$.
c. $40 \div \tan 20^\circ =$ **109.9 m** Use the same method with 20°.
d. $188.2 - 109.9 =$ **78.3 m** Both boats are in the same direction, so subtract the distances.
e. The boat at 20° is **closer**: a larger angle of depression means a nearer boat. The closer the boat, the steeper the line of sight.

Answer to Question 19

a. $\tan\theta = \dfrac{2.4}{4} = 0.6$, so $\theta = 31.0^\circ$ Half the span is 4 m, which is adjacent to the roof angle, and the rise of 2.4 m is opposite.
b. $\sqrt{4^2 + 2.4^2} = \sqrt{21.76} =$ **4.66 m** The rafter is the hypotenuse of the triangle with sides 4 m and 2.4 m.
c. Area $= 2 \times 4.66 \times 10 = 93.3$ m². Cost $= 93.3 \times 14 =$ **£1 306** (to the nearest pound). The roof has two identical rectangles; multiply the total area by the price per square metre.
d. Rise $= 4\tan 40^\circ =$ **3.36 m** The half-span of 4 m is adjacent to the 40° angle, so multiply by $\tan 40^\circ$.

Answer to Question 20

a. $\sin 45^\circ = \dfrac{1}{\sqrt{2}} = 0.7071\ldots$, which matches the calculator. In the isosceles triangle the side opposite 45° is 1 and the hypotenuse is $\sqrt{2}$.
b. In the half-triangle the hypotenuse is 2, the short side 1 and the height $\sqrt{3}$. So $\sin 30^\circ = \dfrac{1}{2}$ and $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$. The 30° angle is at the top of the half-triangle, so the side of 1 is opposite it and $\sqrt{3}$ is adjacent.
c. $\dfrac{\sin\theta}{\cos\theta} = \dfrac{5/13}{12/13} = \dfrac{5}{12} = \tan\theta$. The 13s cancel when you divide one fraction by the other.
d. $\left(\dfrac{5}{13}\right)^2 + \left(\dfrac{12}{13}\right)^2 = \dfrac{25 + 144}{169} = \dfrac{169}{169} = 1$. This identity comes from Pythagoras' theorem divided by the square of the hypotenuse.
e. $\cos^2\theta = 1 - 0.36 = 0.64$, so $\cos\theta =$ **0.8**. The angle is acute, so take the positive square root of 0.64.