Answers - Statistics: averages and grouped data

Year: Year 9

Answer to Question 1

a. Ordered: 3, 4, 7, 7, 9. Mean $= 30 \div 5 =$ **6**, median **7**, mode **7**, range $9 - 3 =$ **6**. The mean is the total divided by how many values there are.
b. Ordered: 10, 11, 12, 14, 15, 18. Mean $= 80 \div 6 =$ **13.3**, median $= (12 + 14) \div 2 =$ **13**, no mode, range $= 18 - 10 =$ **8**. With six values the median is halfway between the 3rd and 4th values.
c. Mean $= 22 \div 5 =$ **4.4**, median **5**, modes **2 and 5**, range **6**. Two values appear twice each, so the data have two modes.
d. Mean, median and mode are all **6**, range **0**. When every value is the same, there is no spread at all.

Answer to Question 2

a. Total $= 40$; known $= 32$; fifth $=$ **8** Total = mean × number of values; subtract the four known numbers. Check: $40 \div 5 = 8$.
b. Total $= 48$; known $= 36$; fourth $=$ **12** Total = 4×12; subtract the three known numbers.
c. Total after 6 tests $= 420$; after 7 tests $= 504$; seventh $=$ **84%** The seventh score is the difference between the two totals: $504 - 420$.
d. The median is the middle value, which is 9, so x must be between 9 and 15, that is $9 \leq x \leq 15$. With x in fourth position, the list stays in order only if x is between 9 and 15.

Answer to Question 3

a. $0 \times 3 + 1 \times 5 + 2 \times 4 + 3 \times 2 + 4 \times 1 = 0 + 5 + 8 + 6 + 4 =$ **23** Multiply each number of goals by its frequency and add the results.
b. $23 \div 15 =$ **1.53** Mean = total goals ÷ number of matches: $23 \div 15 = 1.533\ldots$.
c. Mode **1** (frequency 5). Median is the 8th value; cumulative frequencies 3, 8: the 8th value is **1**. There are 15 values, so the median is the $\frac{15+1}{2} = 8$th value.
d. $4 - 0 =$ **4** Range = largest value − smallest value.

Answer to Question 4

a. The **mode**: the most common size sells most often. The shop needs the size that

appears most often, not a calculated value such as 5.7.

b. The **median**: the very large salary distorts the mean. One very large salary pulls the mean far above the typical salary.
c. The **mean**: every mark counts equally. All the marks are on the same scale and there is no extreme value, so the mean uses all the information.
d. Mean $= 374 \div 6 =$ **£62.3k**; median $= (25 + 26) \div 2 =$ **£25.5k**. The median is much more representative. The total of the salaries is £374k; the median of six values is halfway between the 3rd and 4th.

Answer to Question 5

a. 10 (frequency 9) The modal class is the class with the highest frequency.
b. Total $= 6 \times 5 + 9 \times 15 + 5 \times 25 = 30 + 135 + 125 = 290$. Mean $\approx 290 \div 20 =$ **14.5 minutes**. Multiply each midpoint by its frequency, add, then divide by the total frequency of 20.
c. The 10th and 11th values: cumulative 6 and 15, so the median is in 10. With 20 values the median is between the 10th and 11th, which lie in the second class.
d. We assume every value is at the midpoint of its class, but the actual times are not known. Using midpoints is a sensible estimate, but the exact values inside each class are unknown.

Answer to Question 6

a. **Team A** (24 is greater than 22). Compare the means: the larger mean is the higher average.
b. **Team A**: its range is smaller, so its scores are closer together. A smaller range means the scores are less spread out.
c. Team A scored more on average and was more consistent than Team B. A good comparison mentions both an average and a measure of spread.
d. The mean rises by 5 to **29** and the range stays **6**, because every score moves by the same amount. Adding the same number to every value moves all of them up without changing the gaps between them.

Answer to Question 7

a. $5, 6, 6, 7, 7, 8, 8, 8, 9, 10$ Write the scores from smallest to largest so the median can be found.
b. Mean $= 74 \div 10 =$ **7.4**; median $= (7 + 8) \div 2 =$ **7.5**; mode **8**; range **5**. Mean: the total is 74. Median: halfway between the 5th and 6th values.
c. Scores above 7: 8, 8, 8, 9, 10 = 5 pupils out of 10, which is half, so "most" is **not** correct. Exactly half the pupils scored more than 7, and "most" means more than half.
d. Total 84, mean $= 84 \div 11 =$ **7.6**. Ordered, the 6th value is **8**, so the median is 8.

There are now 11 values, so the median is the 6th value in the ordered list.

Answer to Question 8

a. Ordered: 2, 3, 4, 5, 5, 5, 10, 20. Mean $= 54 \div 8 =$ **£6.75**; median $= (5 + 5) \div 2 =$ **£5**; mode **£5**. Order the values first; the total is 54 and there are 8 values.
b. The **£20** is much larger than the others. £20 is far bigger than every other value, so it is an outlier.
c. $34 \div 7 =$ **£4.86** (to the nearest penny). Remove the £20 from the total and divide by the 7 remaining pupils.
d. The **median (£5)** or mode: the mean is pulled up by the £20. The median and mode are not affected by the single large value.

Answer to Question 9

a. $6 + 10 + 9 + 8 =$ **33** Multiply each number of goals by its frequency and add.
b. $33 \div 20 =$ **1.65** Mean = total goals ÷ 20 matches.
c. Mode **1**. Median: 10th and 11th values are 1 and 2 (cumulative 4, 10, 15), so median $= 1.5$. Range **4**. The mode has the highest frequency; the median is between the 10th and 11th values.
d. Needed total $= 50$, so $50 - 33 =$ **17 goals** in the next 5 matches. A mean of 2 over 25 matches needs a total of $2 \times 25 = 50$ goals.

Answer to Question 10

a. Ordered: 12, 13, 14, 15, 16, 17, 18. Mean $= 105 \div 7 =$ **15**, median **15**, range **6**. Order the temperatures; the total is 105 and there are 7 values.
b. New total $= 127$, mean $= 127 \div 8 =$ **15.9°C** Add 22 to the total and divide by 8 days.
c. New ordered list has 8 values; the median is $(15 + 16) \div 2 =$ **15.5**. With 8 values the median is halfway between the 4th and 5th values.
d. 22°C is 4 degrees above the previous maximum of 18, so it is **unusually warm** for these days. Compare 22°C with the range of the other temperatures.

Answer to Question 11

a. 5 The modal class has the highest frequency, 14.
b. Total $= 8 \times 2.5 + 14 \times 7.5 + 6 \times 12.5 + 2 \times 17.5 = 20 + 105 + 75 + 35 = 235$. Mean $\approx 235 \div 30 =$ **7.8 minutes**. Multiply each midpoint by its frequency, add, then divide by 30.
c. 15th and 16th values: cumulative 8, 22 so the median class is 5. With 30 values the median is between the 15th and 16th; the cumulative frequency reaches 22

in the second class.

d. $\frac{6+2}{30} = \frac{8}{30} = \frac{4}{15}$ Add the frequencies of the last two classes and simplify the fraction.

Answer to Question 12

a. **9A** (62% is more than 58%). Compare the two means.
b. **9B**: its range is smaller, so the marks are less spread out. The range measures spread; the smaller range shows more consistent marks.
c. For: higher mean. Against: much bigger range, so some pupils did poorly. The range is influenced by extreme marks. A fair comparison uses both the average and the spread.
d. Highest $= 25 + 18 =$ **43%**, because the range is the highest minus the lowest. Range = highest − lowest, so highest = lowest + range.

Answer to Question 13

a. Ordered: 12, 15, 15, 15, 18, 20, 22. Mean $= 117 \div 7 =$ **16.7**, median **15**, mode **15**. Order the heights; the total is 117 and there are 7 plants.
b. $22 - 12 =$ **10** Range = tallest − shortest.
c. New total $= 117 - 12 + 30 = 135$, mean $= 135 \div 7 =$ **19.3** Take 12 away from the total and add 30, then divide by 7.
d. The mean rose from 16.7 to 19.3 (by 2.6). The median rose from 15 to 18 (by 3). The **mode** stayed at 15. The mode is the least affected by the change. The new ordered list is 15, 15, 15, 18, 20, 22, 30, so the middle value is 18.

Answer to Question 14

a. Size **6** Size 6 has the highest frequency, 12.
b. Total $= 12 + 35 + 72 + 56 + 40 + 18 = 233$. Mean $= 233 \div 37 =$ **6.3** Multiply each size by its frequency, add, then divide by 37.
c. The 19th value: cumulative 3, 10, 22, so the median is size **6**. With 37 values the median is the 19th value.
d. The **mode**: it shows which size is bought most often, and the mean (6.3) is not a size that exists. The shop must stock real sizes, so a whole-number average about popularity is most useful.

Answer to Question 15

a. Friends: $5 \times 15 - 4 \times 14 = 75 - 56 =$ **19**. Numbers: total 52; $52 - 4 = 48$; $48 \div 7 =$ **6.86**. Marks: $6 \times 16 - 5 \times 15 = 96 - 75 =$ **21**. In each case, find the old total and

the new total with total = mean × number of values, then subtract.

b. The mean is a balancing point of the data, so it cannot be bigger than the largest value or smaller than the smallest. If every value were moved to the same number, that number would be the mean; it cannot go outside the data.

c. Total for mean 20 of 5 friends is 100, so the fifth would be $100 - 56 = 44$, which is possible mathematically, though a 44-year-old friend is not very realistic. The answer shows that one extreme value can change the mean a lot.

Answer to Question 16

a. $160 \leq h$ The modal class has the highest frequency, 18.

b. Total $= 4 \times 145 + 12 \times 155 + 18 \times 165 + 6 \times 175 = 580 + 1860 + 2970 + 1050 = 6460$. Mean $\approx 6460 \div 40 =$ **161.5 cm**. Multiply each midpoint by its frequency, add, then divide by 40.

c. The 20th and 21st values: cumulative 4, 16, 34. The median lies in $160 \leq h$. With 40 values the median is between the 20th and 21st; the cumulative frequency passes 20 in the third class.

d. $4 + 12 =$ **16 pupils** Add the frequencies of the first two classes.

e. Actual range $= 178 - 142 =$ **36 cm**. The greatest possible range from the table is $180 - 140 = 40$ cm. The class limits only show the widest the data could possibly be; the actual range uses the real tallest and shortest pupils.

Answer to Question 17

a. **Shop A**: both the mean (25 against 27) and the median (22 against 26) are lower. Lower delivery times are better, so compare both averages.

b. **Shop B**: its range is only 10 minutes, so times are very similar. A small range means that delivery times are predictable.

c. Shop B: with a median of 26 and a range of 10 it is likely to be within 30 minutes. Shop A could take up to 45 minutes. Reliability matters more than a slightly faster average when there is a strict time limit.

d. $45 - 30 =$ **15 minutes** Range = slowest − fastest, so fastest = slowest − range.

e. The mean is greater than the median, so a few very long deliveries pull the mean up (the data are skewed). A mean bigger than the median shows that a few large values are pulling the mean up.

Answer to Question 18

a. $6 + x + 5 + 2 = 13 + x$ Add all the frequencies, including the unknown x.

b. Total siblings $= 0 \times 6 + 1 \times x + 2 \times 5 + 3 \times 2 = x + 16$ Multiply each number of brothers and sisters by its frequency.

c. $\frac{x+16}{13+x} = 1.2$, so $x + 16 = 15.6 + 1.2x$, $0.4 = 0.2x$, $x =$ **2**. Multiply both sides by $13 + x$, then collect the x terms. Check: $18 \div 15 = 1.2$.
d. Frequencies 6, 2, 5, 2 (total 15). Mode **0**. The 8th value: cumulative 6, 8 so the median is **1**. The mode has the highest frequency; the median of 15 values is the 8th value.
e. $13 + 2 =$ **15 pupils** Substitute $x = 2$ into $13 + x$.

Answer to Question 19

a. Total $= 417$. Mean $= 417 \div 9 =$ **46.3 min**, median **45 min**, range $60 - 40 =$ **20 min**. Add the nine times; the median is the 5th value.
b. New total 357, mean $357 \div 8 =$ **44.6 min**; median $= (44 + 45) \div 2 =$ **44.5 min**. Remove 60 from the total and divide by 8; the median is now between the 4th and 5th values.
c. The **mean** changed from 46.3 to 44.6 (a drop of 1.7), the median by only 0.5, because the mean is affected by the extreme value of 60. An extreme value has a big effect on the mean but little effect on the median.
d. $0.625 \times 60 \approx 38$, so **44 min 38 s**. Multiply the decimal part of a minute by 60 to change it into seconds.
e. The **median (45 min)**. The median is not affected by the slow runner.

Answer to Question 20

a. A: $20 \times 62 = 1240$. B: $30 \times 72 = 2160$. Total = mean × number of pupils.
b. $\frac{1240 + 2160}{50} = \frac{3400}{50} =$ **68** Add the two totals and divide by the total number of pupils, 50.
c. There are more pupils in B (30 against 20), so B has more weight in the combined mean. The combined mean is a weighted mean, so the larger group pulls it towards its own mean.
d. $\frac{1240 + 80x}{20 + x} = 68$, so $1240 + 80x = 1360 + 68x$, $12x = 120$, $x =$ **10 pupils**. Write the new total and the new number of pupils in terms of x, then solve the equation.
e. $(2160 - 90) \div 29 = 2070 \div 29 =$ **71.4** Remove the 90 from the total and divide by the 29 pupils who are left.