Lesson - Solving linear equations and rearranging formulae

Year: Year 9

This lesson covers solving linear equations (including brackets, unknowns on both sides and fractions), forming equations, and changing the subject of a formula.

Starter activity

A balance has two bags of marbles and three marbles on the left, and one bag and eight marbles on the right. The balance is level. Each bag holds the same number of marbles. How can you find how many are in a bag? Try taking the same thing off both sides.

Equations and the balance idea

Definition: equation, solving and inverse operations

An equation is a statement that two expressions are equal, such as $3x + 5 = 20$.
Solving means finding the value of the unknown that makes the statement true.
Inverse operations (the opposite operation) undo what has been done to x.

Rule: keep the balance

Think of a pair of balanced scales: whatever you do to one side you must do to the other, so that the scales stay level.

One- and two-step equations

Worked example 1: two steps

Solve $3x + 5 = 20$.

Subtract 5 from both sides: $3x = 15$. Divide both sides by 3: $x = 5$. Check: $3 \times 5 + 5 = 20$. Correct.

Worked example 2: a negative coefficient

Solve $7 - 2x = 15$.

Subtract 7: $-2x = 8$. Divide by -2: $x = -4$. Check: $7 - 2 \times (-4) = 7 + 8 = 15$. Correct.

Equations with brackets

Method: equations with brackets

1. Expand the brackets (or divide both sides by the number outside).
2. Collect the x terms on one side and the numbers on the other.
3. Solve and check in the original equation.

Worked example 3: a bracket

Solve $3(x-2)=18$.

Expand: $3x-6=18$. Add 6: $3x=24$. Divide by 3: $x=8$. (A shortcut is to divide both sides by 3 first, giving $x-2=6$.)

Unknowns on both sides

Key fact: collecting terms

Collect the x terms on one side and the numbers on the other. It is neatest to move the smaller x term so that you keep a positive coefficient.

Worked example 4: both sides

Solve $5x+3=2x+15$.

Subtract $2x$: $3x+3=15$. Subtract 3: $3x=12$. So $x=4$. Check: left $=23$, right $=23$. Correct.

Worked example 5: brackets on both sides

Solve $2(3x+1)=4(x+5)$.

Expand: $6x+2=4x+20$. Subtract $4x$: $2x+2=20$. Subtract 2: $2x=18$. So $x=9$.

Equations with fractions

Rule: clearing fractions

To remove fractions, multiply every term on both sides by the lowest common multiple of the denominators.

Worked example 6: one fraction

Solve $\frac{x}{4}+3=8$.

Subtract 3: $\frac{x}{4} = 5$. Multiply by 4: $x = 20$.

Worked example 7: fractions on both sides

Solve $\frac{2x-1}{3} = \frac{x+4}{2}$.

Multiply both sides by 6 (the LCM of 3 and 2): $2(2x-1) = 3(x+4)$. Expand: $4x - 2 = 3x + 12$. Subtract $3x$: $x - 2 = 12$. So $x = 14$.

Forming and solving equations

Worked example 8: angles in a triangle

The angles of a triangle are x, $x + 10$ and $2x - 10$ degrees. Find them.

Choose a letter for the unknown, write an equation, solve it and check. The angles add to 180°, so $x + (x + 10) + (2x - 10) = 180$. Simplify: $4x = 180$, so $x = 45$. The angles are 45°, 55° and 80°, which add to 180°.

Formulae and changing the subject

Definition: formula and subject

A **formula** is a rule that links quantities, such as $A = \frac{1}{2}bh$ for the area of a triangle, $v = u + at$ for speed or $F = \frac{9}{5}C + 32$ for converting temperatures. The **subject** of a formula is the letter on its own on one side.

Method: changing the subject

1. Treat the formula like an equation.
2. Use inverse operations, doing the same to both sides.
3. Keep a whole side together over one fraction line.
4. Check by substituting numbers.

Worked example 9: making x the subject

Make x the subject of $y = 3x + 2$.

Subtract 2: $y - 2 = 3x$. Divide by 3: $x = \frac{y-2}{3}$.

Worked example 10: making t the subject

Make t the subject of $v = u + at$, then find t when $v = 7$, $u = 3$, $a = 2$.

Subtract u: $v - u = at$. Divide by a: $t = \dfrac{v-u}{a}$. With the numbers: $t = \dfrac{7-3}{2} = 2$.

Worked example 11: making h the subject

Make h the subject of $A = \frac{1}{2}bh$.

Multiply by 2: $2A = bh$. Divide by b: $h = \dfrac{2A}{b}$.

Watch out: common mistakes

- Doing an operation to only one side of the equation.
- Forgetting to multiply *every* term when clearing fractions.
- Dividing only the first term: from $3x = 15 - 6$ you must divide the whole of the other side, not just one term.
- Sign errors when dividing by a negative number.
- When changing the subject, not dividing the *whole* side: $t = \dfrac{v-u}{a}$, not $v - \dfrac{u}{a}$.

Worked example 12: an equation with negative terms on both sides

Solve $3 - 2x = 11 + 2x$.

Add $2x$ to both sides: $3 = 11 + 4x$. Subtract 11: $-8 = 4x$. Divide by 4: $x = -2$. Check: left $= 3 + 4 = 7$, right $= 11 - 4 = 7$.

Worked example 13: a formula with a fraction

Make h the subject of $A = \dfrac{1}{2}(a+b)h$.

Multiply both sides by 2: $2A = (a+b)h$. Divide by $(a+b)$: $h = \dfrac{2A}{a+b}$. For $A = 30$, $a = 4$, $b = 6$: $h = 60 \div 10 = 6$.

Worked example 14: solving then substituting

Given $y = 3x + 4$, find x when $y = 19$, and find y when $x = -2$.

$19 = 3x + 4$, so $3x = 15$ and $x = 5$. When $x = -2$: $y = 3(-2) + 4 = -2$.

Worked example 15: an equation from a word problem

A piece of ribbon 90 cm long is cut in two. One piece is 14 cm longer than the other. How long is each?

Let the short piece be x cm. $x + (x + 14) = 90$, so $2x = 76$ and $x = 38$. The pieces are

38 cm and 52 cm.

Note: show every step

Write each line of working under the one before, and do the same operation to both sides, writing it at the side of the equation if it helps. This makes it easy to find an error. Always check by substituting your answer back into the *original* equation, not into a line you have already changed. If the two sides do not match, go back through your lines one at a time and look for a sign error, which is the most common mistake in equations. Finding your own mistakes is a key skill.

Watch out: negatives and fractions

- When you move a term to the other side its sign changes. Subtracting $2x$ from both sides is not the same as dropping it.
- When you multiply to clear fractions, multiply *every* term, including whole numbers: $\frac{x}{2} + 3 = 5$ becomes $x + 6 = 10$.
- When you divide by a negative number, the answer changes sign: $-3x = 12$ gives $x = -4$.

Did you know?

The equals sign = was first used in 1557 by a Welsh mathematician called Robert Recorde. He wrote that he chose two parallel lines of the same length "because no two things can be more equal". Before that, mathematicians wrote the word for equals in full, in Latin.

From words to equations

Method: forming an equation from a problem

1. Read the whole problem and decide what you are trying to find. Give it a letter, such as x, and write down what the letter stands for, including the units.
2. Write every other quantity in terms of that letter. For example, "three more than" becomes $x + 3$ and "twice as many" becomes $2x$.
3. Look for a fact that gives two expressions with the same value, such as a total, a perimeter or an angle sum. Put an equals sign between them.
4. Solve the equation step by step, doing the same to both sides.
5. Answer the question that was asked, in words, and check the answer in the original problem, not just in your equation.

Worked example 16: consecutive numbers

Three consecutive whole numbers add up to 87. Find the numbers.

Let the smallest number be n. The next two numbers are $n+1$ and $n+2$, because consecutive numbers go up by one each time. Their total is $n+(n+1)+(n+2)=3n+3$, so the equation is $3n+3=87$. Subtract 3 from both sides to get $3n=84$, then divide by 3 to get $n=28$. The numbers are 28, 29 and 30. Check: $28+29+30=87$, so the answer is correct.

Worked example 17: a subject with a minus sign

Make x the subject of $y=12-3x$.

The term containing x is negative, so it is easiest to add $3x$ to both sides first: $y+3x=12$. Next subtract y from both sides: $3x=12-y$. Finally divide both sides by 3 to get $x=\dfrac{12-y}{3}$. Check with a number: if $x=2$, then $y=12-6=6$, and the new formula gives $x=\dfrac{12-6}{3}=2$, as it should.

Note: equation, formula, identity and expression

An **expression** such as $3x+2$ has no equals sign. An **equation** such as $3x+2=14$ is true only for particular values of the letter, which you find by solving it. A **formula** such as $A=lw$ links two or more quantities and works for any values you substitute. An **identity** such as $2(x+3)=2x+6$ is true for every value of x. Using the right word helps you to understand what a question is asking you to do. "Solve" goes with an equation, "substitute" and "rearrange" go with a formula, "simplify" goes with an expression, and "show that" or "prove" usually goes with an identity. If you try to solve an identity, every number works, and if you try to solve an equation with no solution, the letters cancel and you are left with something false, such as $3=5$.

Otto's tip

Always check your solution by substituting it back into the *original* equation. If both sides match, you are done. Hoot hoot!