Answers - Solving linear equations and rearranging formulae

Year: Year 9

Answer to Question 1

a. $3x = 15$, so $x =$ **5**. Check: $15 + 5 = 20$. Subtract 5 from both sides, then divide both sides by 3.
b. $4x = 24$, so $x =$ **6**. Check: $24 - 7 = 17$. Add 7 to both sides, then divide both sides by 4.
c. $-2x = 6$, so $x = -3$. Check: $5 + 6 = 11$. Subtract 5 from both sides; dividing by -2 gives a negative answer.
d. $\frac{x}{3} = 5$, so $x =$ **15**. Check: $5 + 4 = 9$. Subtract 4 from both sides, then multiply both sides by 3 to undo the division.

Answer to Question 2

a. $3x + 12 = 27$, $3x = 15$, $x =$ **5** Multiply 3 into both terms of the bracket, subtract 12, then divide by 3. Check: $3 \times 9 = 27$.
b. $6x - 2 = 22$, $6x = 24$, $x =$ **4** Expand to $6x - 2$, add 2 to both sides, then divide by 6. Check: $2 \times 11 = 22$.
c. $4x - 8 + 3 = 19$, $4x - 5 = 19$, $4x = 24$, $x =$ **6** Expand and collect the numbers $-8 + 3 = -5$ before solving. Check: $4 \times 4 + 3 = 19$.
d. $10x - 15 = 4x + 12$, $6x = 27$, $x =$ **4.5** Expand both sides, subtract $4x$ and add 15. Check: $5 \times 6 = 30$ and $4 \times 7.5 = 30$.

Answer to Question 3

a. $3x = 15$, so $x =$ **5** Subtract $2x$ from both sides and subtract 3. Check: $28 = 28$.
b. $4x = 16$, so $x =$ **4** Subtract $3x$ and add 4 to both sides. Check: $24 = 24$.
c. $10 = 2x$, so $x =$ **5** Move the smaller x term: subtract $4x$ and add 1 to both sides. Check: $29 = 29$.
d. $3x + 6 = x + 14$, $2x = 8$, $x =$ **4** Expand the bracket first, then subtract x and 6 from both sides. Check: $3 \times 6 = 18 = 4 + 14$.

Answer to Question 4

a. $x = 7 \times 4 =$ **28** Multiply both sides by 4 to undo the division.
b. $x + 3 = 16$, $x =$ **13** Multiply both sides by 2, then subtract 3. Check: $16 \div 2 = 8$.
c. $2x - 1 = 15$, $2x = 16$, $x =$ **8** Multiply both sides by 3, add 1, then divide by 2.

Check: $15 \div 3 = 5$.

d. Multiply by 6: $3x + 2x = 60$, $5x = 60$, $x =$ **12** The lowest common multiple of 2 and 3 is 6, so multiply every term by 6. Check: $6 + 4 = 10$.

Answer to Question 5

a. Subtract 5: $x = y - 5$ Undo adding 5 by subtracting 5 from both sides.

b. Divide by 3: $x = \frac{y}{3}$ Undo multiplying by 3 by dividing both sides by 3.

c. Add 4 then divide by 2: $x = \frac{y + 4}{2}$ Undo the operations in reverse order: first add 4, then divide by 2.

d. Divide by w: $l = \frac{A}{w}$ l is multiplied by w, so divide both sides by w.

Answer to Question 6

a. $3 + 2 \times 5 =$ **13** Multiply a by t before adding u.

b. $\frac{1}{2} \times 10 \times 3 =$ **15** Add the parallel sides first: $4 + 6 = 10$, then multiply by $\frac{1}{2}$ and by 3.

c. $2 \times \pi \times 5 =$ **31.4** Use the π key: $10\pi = 31.415\ldots$, which rounds to 31.4.

d. $1.8 \times 15 + 32 = 27 + 32 =$ **59** Multiply 15 by 1.8 first, then add 32.

Answer to Question 7

a. $3x - 7 = 20$, $3x = 27$, $x =$ **9** Turn the words into an equation, add 7 to both sides and divide by 3.

b. $2x + 5 = x + 12$, $x =$ **7** The answer is 12 more than her number, so the right-hand side is $x + 12$. Subtract x and 5 from both sides.

c. $5(x + 4) = 60$, $x + 4 = 12$, $x =$ **8** Adding 4 comes before multiplying, so put $x + 4$ in a bracket. Divide both sides by 5 first.

d. Ali: $27 - 7 = 20$. Bea: $14 + 5 = 19 = 7 + 12$. Cal: $(8 + 4) \times 5 = 60$. All correct. Substituting each answer back into the words gives the stated result.

Answer to Question 8

a. $2(2x + 3) + 2x = 36$, so $4x + 6 + 2x = 36$, $6x = 30$, $x =$ **5**. The perimeter is two lengths plus two widths. Expand, collect like terms and solve.

b. Width $= 5$ m. Length $= 2 \times 5 + 3 =$ **13 m**. Substitute $x = 5$ into the expressions for the length and the width.

c. $13 \times 5 =$ **65 m²** Area of a rectangle = length × width.

d. $65 \times 4.50 =$ **£292.50** Multiply the area by the price per square metre.

Answer to Question 9

a. $x + 2x + 10 + 3x - 10 = 180$, so $6x = 180$ and $x =$ **30**. The angles in a triangle add up to $180°$. The numbers $+10$ and -10 cancel.
b. $30°$**,** $70°$**,** $80°$ Substitute $x = 30$ into each expression.
c. $30 + 70 + 80 = 180$. The three angles add up to exactly $180°$, so the answer is correct.
d. **Scalene**: all three angles are different, so no sides are equal. In a triangle, equal angles are opposite equal sides, so different angles mean different sides.

Answer to Question 10

a. A: $3 + 1.5m$. B: $1 + 2m$. Each cost is a fixed charge plus the price per mile multiplied by the number of miles.
b. $3 + 1.5m = 1 + 2m$, so $2 = 0.5m$ and $m =$ **4 miles**. Set the two expressions equal, then subtract $1.5m$ and 1 from both sides.
c. $3 + 1.5 \times 4 =$ **£9** Substitute $m = 4$ into either expression; taxi B gives $1 + 8 = 9$ too.
d. A: $3 + 15 = £18$; B: $1 + 20 = £21$. Taxi **A** is cheaper by **£3**. Substitute $m = 10$ into both expressions and compare.

Answer to Question 11

a. Child tickets: $40 - a$ There are 40 tickets altogether, so the rest of them are child tickets.
b. $8a + 5(40 - a) = 260$, so $8a + 200 - 5a = 260$, $3a = 60$, $a =$ **20**. Adult tickets raise $8a$ pounds and child tickets raise $5(40 - a)$ pounds; expand and solve.
c. **20 adult** tickets and **20 child** tickets. Check: $160 + 100 = 260$. Substitute $a = 20$ into $40 - a$ and check the total money.
d. The most it could be is 40 adult tickets: $40 \times 8 = £320$, so £300 is possible, with $3a + 200 = 300$, $a = 33.3$, which is not a whole number, so **not exactly £300**. The total $3a + 200$ can only be 299 or 302 near £300, because a must be a whole number.

Answer to Question 12

a. $40 \times 5 + 20 = 220$ min $=$ **3 hours 40 minutes** Substitute $w = 5$, then change minutes into hours and minutes.
b. $T - 20 = 40w$, so $w = \dfrac{T - 20}{40}$ Subtract 20 from both sides, then divide both sides by 40.
c. $w = (300 - 20) \div 40 =$ **7 kg** Substitute $T = 300$ into the rearranged formula.
d. $T = 260$ min $= 4$ h 20 min. $17{:}30 - 4{:}20 =$ **13:10**. Find the cooking time for 6 kg,

then count back from 17:30.

Answer to Question 13

a. $t = \frac{d}{s}$ and $s = \frac{d}{t}$ Divide both sides of $d = st$ by s to find t, or by t to find s.

b. $1800 \div 600 =$ **3 hours** Substitute into $t = \frac{d}{s}$.

c. $1800 \div 2.5 =$ **720 km/h** Substitute into $s = \frac{d}{t}$.

d. $3 \times 3000 =$ **9 000 litres** The first flight takes 3 hours, so multiply the fuel per hour by 3.

Answer to Question 14

a. Alex gets **£**$(x + 3)$. Alex gets 3 pounds more than Sam.
b. $x + x + 3 = 17$, $2x = 14$, $x =$ **7**. Add the two amounts and set the total equal to 17, then subtract 3 and divide by 2.
c. Sam £7, Alex **£10**. Sam gets $x = 7$ pounds and Alex gets $7 + 3$ pounds.
d. $8 \times 17 =$ **£136** Together they receive £17 a week, so multiply by 8.

Answer to Question 15

a. $5x = 25$, $x =$ **5**. Check: $32 = 32$. Subtract $2x$ from both sides and add 3.
b. $6x + 3 = 4x + 15$, $2x = 12$, $x =$ **6**. Check: $39 = 39$. Expand the bracket, then subtract $4x$ and 3 from both sides.
c. $x + 5 = 3x - 3$, $8 = 2x$, $x =$ **4**. Check: $3 = 3$. Multiply both sides by 3 first, then collect the x terms on the right.
d. $2x - 6 - 3x + 12 = 5$, $-x + 6 = 5$, $x =$ **1**. Check: $-4 + 9 = 5$. Take care with $-3(x - 4) = -3x + 12$; then collect like terms.
e. Equation (c). Equation (c) has a denominator of 3, which must be cleared first.

Answer to Question 16

a. $a = \frac{v - u}{t}$; $l = \frac{P}{2} - w$; $h = \frac{2A}{b}$; $x = \frac{y - c}{m}$. Undo each operation in reverse order. For example, in $P = 2(l + w)$ divide by 2 and then subtract w.
b. $a = 20 \div 4 =$ **5**; $l = 15 - 6 =$ **9**; $h = 60 \div 12 =$ **5**; $x = 12 \div 4 =$ **3**. Substitute the given values into each rearranged formula.
c. Check A: $\frac{1}{2} \times 12 \times 5 = 30$. Correct. Substituting the answer back into the original formula gives the given value.
d. If $b = 0$ then A would always be 0, and division by zero is impossible. The formula

$h = \dfrac{2A}{b}$ would need a division by zero.

Answer to Question 17

a. A: $15 + 0.02m$. B: $5 + 0.05m$. Change pence into pounds: 2p is £0.02 and 5p is £0.05.
b. $15 + 0.02m = 5 + 0.05m$, so $10 = 0.03m$ and $m = 333.3$, so about **333 minutes**. Set the costs equal, subtract $0.02m$ and 5 from both sides, then divide by 0.03.
c. 400 minutes: A is £23 and B is £25, so **A** is cheaper. 100 minutes: **B** is cheaper (£10 against £17). Above about 333 minutes Plan A is cheaper; below it Plan B is cheaper.
d. 250 minutes: A $= 15 + 5 = £20$; B $= 5 + 12.5 = £17.50$. Plan B saves $2.50 \times 12 =$ **£30** a year. Plan B is £2.50 cheaper each month, and there are 12 months in a year.

Answer to Question 18

a. $212 = 1.8C + 32$, $180 = 1.8C$, $C =$ **100** Subtract 32 from both sides, then divide by 1.8.
b. $-4 - 32 = -36$, $C = -36 \div 1.8 = -20$ Subtract 32 from -4 carefully, then divide by 1.8.
c. $F - 32 = 1.8C$, so $C = \dfrac{F - 32}{1.8}$ Subtract 32 from both sides, then divide both sides by 1.8.
d. Put $F = C$: $C = 1.8C + 32$, so $-0.8C = 32$ and $C = -40$. Replace F by C, collect the C terms on one side and solve.
e. At -40, $1.8 \times (-40) + 32 = -72 + 32 = -40$, so both scales read -40. Substituting -40 into the formula gives -40 back.

Answer to Question 19

a. Length $= x + 20$ The length is 20 m more than the width.
b. $2(x + x + 20) = 140$, so $4x + 40 = 140$, $x =$ **25**. The perimeter is twice the length plus twice the width; expand and solve.
c. Width 25 m, length **45 m**, area $25 \times 45 =$ **1 125 m²**. Substitute $x = 25$, then multiply the length by the width.
d. $140 \times 6.50 =$ **£910** Multiply the perimeter by the price per metre.
e. $137 \times 6.50 =$ **£890.50** The gate means 3 m less fence: $140 - 3 = 137$ m.

Answer to Question 20

a. $4b + 6 = 3(b + 6)$, so $4b + 6 = 3b + 18$ and $b =$ **12**. In 6 years both people are 6 years older, so write $4b + 6$ and $b + 6$.
b. Mother is $4 \times 12 =$ **48** now and **54** in 6 years, which is 3×18. Substitute $b = 12$; in 6 years Ben is 18 and his mother is 54.
c. Multiply by 6: $3(3x + 1) - 2(x - 2) = 21$, so $9x + 3 - 2x + 4 = 21$, $7x = 14$, $x =$ **2**. The lowest common multiple of 2 and 3 is 6, so multiply every term by 6 and expand carefully.
d. $\frac{7}{2} - 0 = \frac{7}{2}$. Correct. Substituting $x = 2$ gives $\frac{7}{2} - \frac{0}{3} = \frac{7}{2}$.
e. $2y = 3x + 1$, $3x = 2y - 1$, so $x = \frac{2y - 1}{3}$. Multiply both sides by 2, subtract 1 and divide by 3.