Answers - Similarity and enlargement

Year: Year 9

Answer to Question 1

a. $4 \times 2.5 = 10$ and $3 \times 2.5 = 7.5$: **10 cm by 7.5 cm** Multiply every side by the scale factor 2.5; the angles of the rectangle do not change.
b. $5 \times 3 =$ **15 cm** Multiply the length by the scale factor 3.
c. $15 \div 6 =$ **2.5** Scale factor = new length ÷ original length.
d. $5 \div 20 =$ **0.25** The scale factor is less than 1, so the shape becomes smaller: each length is a quarter of the original.

Answer to Question 2

a. Scale factor $9 \div 3 = 3$, so $x = 5 \times 3 =$ **15** Divide a pair of corresponding sides to find the scale factor, then multiply the third side. This is the 3, 4, 5 triangle enlarged.
b. Scale factor $9 \div 6 = 1.5$, so $y = 14 \times 1.5 =$ **21** Corresponding sides are in the same order: $9 \div 6 = 1.5$ and $15 \div 10 = 1.5$ agree.
c. Scale factor $12 \div 8 = 1.5$, so $z = 5 \times 1.5 =$ **7.5 cm** The longer sides correspond, so the scale factor is $12 \div 8$.
d. Scale factor $10.5 \div 7 = 1.5$, so $w = 4 \times 1.5 =$ **6** The two equal sides correspond, so the scale factor is $10.5 \div 7$.

Answer to Question 3

a. Ratios $2, 2, 2$: **similar** but not congruent. Each side is multiplied by 2, so the shape is the same but bigger.
b. All sides equal: **congruent**. The three sides are identical, so the triangles are exactly the same size and shape.
c. Ratios $2, 2, 2.25$ are not equal: **neither**. For similar shapes, all three ratios of corresponding sides must be equal.
d. All squares are similar: **similar**, not congruent (different sizes). Every square has four right angles and equal sides, so any two squares are similar.

Answer to Question 4

a. $A'(2, 2)$, $B'(6, 2)$, $C'(2, 4)$ With the centre at the origin, multiply each coordinate by 2.
b. $A'(3, 3)$, $B'(9, 3)$, $C'(3, 6)$ Multiply each coordinate by 3.
c. $P'(2, 3)$ and $Q'(4, 1)$ Multiply each coordinate by $\frac{1}{2}$; the image is smaller and

closer to the origin.

d. Original: $\frac{1}{2} \times 2 \times 1 = 1$. Image: base 4, height 2, so $\frac{1}{2} \times 4 \times 2 = 4$. The area is multiplied by $2^2 = \mathbf{4}$. Lengths are multiplied by 2, so the area is multiplied by $2 \times 2 = 4$.

Answer to Question 5

a. Vector $(2, 1)$, doubled $(4, 2)$: $P'(5, 3)$ The vector from the centre is $(3 - 1, 2 - 1)$; multiply it by 2 and add it back to $(1, 1)$.

b. Vector $(1, 3)$, doubled $(2, 6)$: $Q'(3, 7)$ The vector from the centre is $(2 - 1, 4 - 1)$; double it and add to the centre.

c. Vector $(2, 1)$, times 3 is $(6, 3)$: $R'(6, 4)$ The vector from $(0, 1)$ to R is $(2, 1)$; multiply by 3 and add to $(0, 1)$.

d. Vector $(8, 4)$, halved $(4, 2)$: $S'(5, 3)$ Halve the vector $(8, 4)$ and add the result to the centre $(1, 1)$.

Answer to Question 6

a. **SSS** Three pairs of equal sides prove congruence by SSS.

b. **SAS** (the angle is between the two sides) Two sides and the included angle are equal, so SAS applies.

c. **ASA** Two angles and the side between them are equal, so ASA applies.

d. **Not enough information**: AAA only shows that the triangles are similar. Equal angles fix the shape but not the size, so the triangles could be different sizes.

Answer to Question 7

a. 4.8 m $= 480$ cm. $480 \div 24 =$ **20 cm** Change metres to centimetres, then divide by 24.

b. $7 \times 24 = 168$ cm $=$ **1.68 m** Real lengths are 24 times the model lengths; divide by 100 to change centimetres into metres.

c. $45 \div 24 =$ **1.875 cm** Change 0.45 m into 45 cm, then divide by 24.

d. $480 \div 12 =$ **40 cm** A 1 : 12 model is twice as big as a 1 : 24 model, so its length is $2 \times 20 = 40$ cm.

Answer to Question 8

a. $12 \div 2 =$ **6** The tree's shadow is 6 times the stick's shadow.

b. $1.5 \times 6 =$ **9 m** The tree's height is also 6 times the stick's height, because the triangles are similar.

c. The scale factor is $3 \div 2 = 1.5$, so the tree's shadow is $12 \times 1.5 =$ **18 m**. Both shadows change by the same scale factor, because the sun's rays are parallel.

d. $1.5 \times (8 \div 2) = 1.5 \times 4 =$ **6 m** At the first time the flagpole's shadow is 4 times the stick's shadow, so its height is 4 times 1.5 m.

Answer to Question 9

a. Scale factor $24 \div 15 = 1.6$. Height $= 10 \times 1.6 =$ **16 cm**. Scale factor = new width ÷ old width; multiply the height by the same factor.
b. $60 \div 15 = 4$, height $= 10 \times 4 =$ **40 cm**. Divide 60 by 15 to find the scale factor, then multiply the height.
c. No: $30 \div 15 = 2$ gives a height of 20 cm, not 25 cm, so the shapes are not similar. For similar rectangles the ratio of the sides must be the same; $30 : 25$ is not the same as $15 : 10$.
d. The width allows scale factor 2 and the height allows 2.5, so the limit is **2**. The print is **30 cm by 20 cm**. The smaller of the two possible scale factors must be used so that the print fits in both directions.

Answer to Question 10

a. 1 km $= 100\,000$ cm, and $100\,000 \div 4 = 25\,000$, so 1 cm is 25 000 cm $= 250$ m. There are 100 000 cm in 1 km, and 4 cm on the map represents this distance.
b. $7.5 \times 250 = 1875$ m $=$ **1.875 km** Each centimetre on the map represents 250 m.
c. $3500 \div 250 =$ **14 cm** Change 3.5 km into 3 500 m, then divide by 250 m per cm.
d. $6 \times 250 = 1500$ m and $4 \times 250 = 1000$ m: **1.5 km by 1 km**, area **1.5 km²**. Change each side to real lengths first, then multiply to find the area.

Answer to Question 11

a. $36 \div 4 =$ **9** The small and large triangles share the angle at Hassan's eye; divide the two horizontal distances.
b. $1.2 \times 9 =$ **10.8 m** Multiply the height of the pole by the scale factor.
c. $10.8 \div 1.2 = 9$ is the scale factor needed, so the distance to the pole is $24 \div 9 =$ **2.67 m**. The tower and pole are unchanged, so the scale factor must still be 9.
d. The angle at his eye is shared and the pole and tower are both vertical (parallel), so all three angles match. Two pairs of equal angles are enough to prove that two triangles are similar.

Answer to Question 12

a. $200 \div 8 =$ **25** Change 2 m into 200 cm before dividing by 8 cm.
b. $5 \times 25 = 125$ cm $=$ **1.25 m** Multiply the height of the logo by 25 and change to metres.
c. $1.2 \times 25 =$ **30 cm** Every length on the logo is multiplied by 25, including the text.

d. Area $= 2 \times 1.25 = 2.5$ m². Cost $= 2.5 \times 6 =$ **£15** The banner is a rectangle 2 m by 1.25 m; multiply its area by the price per square metre.

Answer to Question 13

a. A4: $297 \div 210 = 1.41$. A3: $420 \div 297 = 1.41$. Divide the long side by the short side for each sheet.
b. Both ratios are about 1.41, so the sheets are **similar**. Similar rectangles have the same ratio of length to width.
c. $420 \div 297 =$ **1.41** (about $\sqrt{2}$). Divide a side of A3 by the corresponding side of A4: the long side of A4 is the short side of A3.
d. $10 \times 1.414 =$ **14.1 cm** Multiply the length by the scale factor of about 1.41.

Answer to Question 14

a. $12 \times 200 = 2400$ cm $=$ **24 m**; $8 \times 200 = 1600$ cm $=$ **16 m**. Every real length is 200 times the plan length; divide by 100 to change centimetres into metres.
b. $2(24 + 16) =$ **80 m** The perimeter is twice the sum of the length and the width.
c. $80 \times 11 =$ **£880** Multiply the perimeter by the price per metre.
d. $1.5 \times 200 = 300$ cm $=$ **3 m** Multiply the plan width by 200 and change to metres.

Answer to Question 15

a. DEF: scale factor 1.5, sides $9, 12, 15$. PQR: scale factor 0.5, sides $3, 4, 5$. Divide a known side by the matching side of ABC to get each scale factor, then multiply the other sides.
b. ABC: 24 cm. DEF: 36 cm. PQR: 12 cm. The perimeters are in the same ratio as the sides. Add the three sides of each triangle; each perimeter is the scale factor times 24 cm.
c. Ratios: $12 \div 6 = 2$, $16 \div 8 = 2$, $21 \div 10 = 2.1$. Not all equal, so **not similar**. For similar triangles all three ratios of corresponding sides must be equal.
d. All of them: $6^2 + 8^2 = 100 = 10^2$, and similar triangles have the same angles, so all three are right-angled. Enlargement does not change angles, so if one triangle has a right angle, all similar ones do.

Answer to Question 16

a. $A'(4, 2)$, $B'(8, 2)$, $C'(4, 8)$ Multiply each coordinate by 2, because the centre is the origin.
b. Vectors from $(1, 1)$ are $(1, 0)$, $(3, 0)$, $(1, 3)$; doubled: $(2, 0)$, $(6, 0)$, $(2, 6)$. Images: $(3, 1)$, $(7, 1)$, $(3, 7)$. Find the vector from the centre to each vertex, double it, and add it back to the centre.

c. Area of $T = \frac{1}{2} \times 2 \times 3 = 3$. Image: $\frac{1}{2} \times 4 \times 6 = 12$. The triangle has base $AB = 2$ and height $AC = 3$; the image has base 4 and height 6.
d. The scale factor is 2 and the area is multiplied by $2^2 = 4$. Area is multiplied by the square of the scale factor.
e. **Enlargement, scale factor 2, centre $(0, 0)$.** To describe an enlargement fully, give the word enlargement, the scale factor and the centre.

Answer to Question 17

a. Scale factor $15 \div 2.4 = 6.25$. Height $= 1.6 \times 6.25 =$ **10 m**. The flagpole's shadow is 6.25 times Ella's shadow, so its height is 6.25 times her height.
b. $45 \div 2.4 = 18.75$. Height $= 1.6 \times 18.75 =$ **30 m**. The building's shadow is 18.75 times Ella's shadow.
c. Scale factor from 2.4 to 1.2 is 0.5, so $15 \times 0.5 =$ **7.5 m**. All shadows are halved at the same time, because the sun's rays are parallel.
d. $45 \times 0.5 =$ **22.5 m** Halve the building's earlier shadow.
e. Around midday the sun is highest, so the rays are steeper and the triangles have a shorter base for the same height. A higher sun means the rays meet the ground at a steeper angle, so every shadow is shorter compared with the object's height.

Answer to Question 18

a. $6 \div 2 =$ **3** Divide the side of cube B by the side of cube A.
b. Surface area $= 6 \times 36 =$ **216 cm²**. Volume $= 6^3 =$ **216 cm³**. Cube B has 6 faces, each $6 \times 6 = 36$ cm², and its volume is $6 \times 6 \times 6$.
c. Area: $216 \div 24 = 9 = 3^2$. Volume: $216 \div 8 = 27 = 3^3$. Areas are multiplied by the square of the scale factor and volumes by its cube.
d. Scale factor $300 \div 20 = 15$. Area ratio $15^2 =$ **225**. Change 3 m into 300 cm, then divide by 20 cm; areas use the square of 15.
e. Paint is proportional to surface area: $40 \times 225 = 9000$ ml $=$ **9 litres**. The paint covers the surface, so it scales with the area factor of 225.

Answer to Question 19

a. Angle A is shared. $DE \parallel BC$, so the corresponding angles at D and B are equal and those at E and C are equal. Three equal angles mean the triangles are similar. Parallel lines make equal corresponding angles, and two triangles with equal angles are similar.
b. $AB = 4 + 2 = 6$, so scale factor $6 \div 4 =$ **1.5** AB corresponds to AD, so divide 6 by 4.
c. $BC = 5 \times 1.5 =$ **7.5 cm** BC corresponds to DE, so multiply 5 by 1.5.
d. $AC = 6 \times 1.5 = 9$, so $EC = 9 - 6 =$ **3 cm**. AC corresponds to AE; subtract AE from AC to find EC.

e. $6 + 7.5 + 9 =$ **22.5 cm** Add the three sides of the large triangle ABC.

Answer to Question 20

a. Short side $= 297 \div 2 = 148.5 \approx$ **149 mm**. Folding A4 in half halves its long side, so A5 is 210 mm by 148.5 mm.

b. A4: $297 \div 210 = 1.414$. A5: $210 \div 148.5 = 1.414$. Both ratios are 1.414, which shows the two sheets are similar.

c. The new sheet has dimensions W and $\frac{L}{2}$. Similar means $\frac{L}{W} = \frac{W}{L/2} = \frac{2W}{L}$. So $\left(\frac{L}{W}\right)^2 = 2$ and $\frac{L}{W} = \sqrt{2}$. Set up the equal ratios and multiply both sides by $\frac{L}{W}$; taking the square root gives $\sqrt{2}$.

d. Scale factor $\sqrt{2} = 1.414$; area ratio $= 2$ (A4 has twice the area of A5). Lengths are multiplied by $\sqrt{2}$, so areas are multiplied by $(\sqrt{2})^2 = 2$.

e. A4 $= 297 \times 210 = 62\,370$ mm$^2 = 0.06237$ m^2, and $1 \div 16 = 0.0625$ m^2. Very close. Each fold halves the area, so A4 is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$ of A0.