Test answers - Sequences and nth term

Year: Year 9 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. The terms go up by 5 each time, so keep adding 5: $19 + 5 =$**24** and $24 + 5 =$**29**. (1 mark)
b. The term-to-term rule is **add 5**, because the difference between consecutive terms is always 5. (1 mark)
c. A common difference of 5 means the nth term starts with $5n$ (1 mark). The sequence $5n$ is 5, 10, 15, 20, and each of our terms is 1 less, so the nth term is $5n - 1$ (1 mark).

Check: $5 \times 1 - 1 = 4$ and $5 \times 4 - 1 = 19$, the first and fourth terms. The term-to-term rule needs the previous term, but the nth term finds any term directly; for example the 100th term is $5 \times 100 - 1 = 499$.

Answer to Question 2 (4 marks)

a. Substitute $n = 1$, 2 and 3 into $4n - 3$: $4 - 3 = 1$, $8 - 3 = 5$, $12 - 3 = 9$. The first three terms are **1, 5, 9**. (1 mark)
b. Substitute $n = 20$: $4 \times 20 - 3 = 80 - 3 =$**77**. (1 mark)
c. Set the formula equal to 99: $4n - 3 = 99$. Add 3: $4n = 102$. Divide by 4: $n = 25.5$ (1 mark). The position must be a whole number, so **no**, 99 is not a term (1 mark).

The 25th term is 97 and the 26th is 101, so 99 falls between them. Another quick check: every term is 1 more than a multiple of 4, and 99 is 3 more than 96, so it cannot appear.

Answer to Question 3 (4 marks)

a. The terms go down by 4, so the common difference is -4 and the nth term contains $-4n$ (1 mark). The term before the first, the zero term, is $25 + 4 = 29$, so the nth term is $29 - 4n$ (1 mark).
b. A term is negative when $29 - 4n < 0$, which gives $4n > 29$ and $n > 7.25$ (1 mark). The first whole number above 7.25 is 8, and the 8th term is $29 - 32 = -3$ (1 mark).

Check (a): $29 - 4 = 25$ and $29 - 16 = 13$, the 1st and 4th terms. The 7th term is $29 - 28 = 1$, still positive, which confirms that the 8th term is the first negative one. Writing $4n + 29$ would be wrong, because the sequence decreases.

Answer to Question 4 (4 marks)

a. Each pattern uses 4 more matchsticks than the one before, so the common difference is 4 and the nth term starts with $4n$ (1 mark). Pattern 1 needs 5, which is $4 + 1$, so the nth term is $4n + 1$ (1 mark).
b. Substitute $n = 25$: $4 \times 25 + 1 =$**101** matchsticks. (1 mark)
c. Set the formula equal to 201: $4n + 1 = 201$, so $4n = 200$ and $n =$**50**. Pattern 50 uses 201 matchsticks. (1 mark)

A typical shape behind $4n + 1$ is a row of n squares sharing sides: one starting stick, then 4 new sticks for each square. Notice that doubling the pattern number from 25 to 50 does not double the sticks, because of the +1.

Answer to Question 5 (4 marks)

a. Divide any term by the one before: $12 \div 3 = 4$, $48 \div 12 = 4$, $192 \div 48 = 4$. The common ratio is **4**. (1 mark)
b. Multiply the last term by the ratio: $192 \times 4 =$**768**. (1 mark)
c. The nth term of this geometric sequence is $3 \times 4^{n-1}$. For $n = 7$: $3 \times 4^6 = 3 \times 4096 =$**12 288**. (1 mark)
d. An arithmetic sequence has a constant difference. Here the differences are 9, 36, 144, which keep growing, so the sequence is not arithmetic. (1 mark)

Check (c) by continuing the list: 768, $768 \times 4 = 3072$, $3072 \times 4 = 12288$ gives the 5th, 6th and 7th terms.