Worksheet - Sequences and nth term

Year: Year 9 · 20

Show your working.

Question 1

Question 1 – Term-to-term rules *(Section A · Skills practice)*

A term-to-term rule tells you how to get from one term to the next. Find the rule and write the next two terms of each sequence.

a. $3, 7, 11, 15, \ldots$
b. $100, 90, 80, 70, \ldots$
c. $2, 6, 18, 54, \ldots$
d. $1, 4, 9, 16, \ldots$

Question 2

Question 2 – Using the nth term *(Section A · Skills practice)*

The nth term formula gives the term in position n. Substitute $n = 1, 2, 3 \ldots$ to find terms.

a. $3n + 2$: write the first four terms and the 10th term.
b. $n^2 + 1$: write the first four terms and the 5th term.
c. $50 - 4n$: write the first three terms and the 12th term.
d. 2^n: write the first five terms.

Question 3

Question 3 – Finding the nth term *(Section A · Skills practice)*

For an arithmetic sequence the nth term is $dn + (a - d)$, where d is the common difference and a is the first term. Find the nth term of each sequence.

a. $5, 8, 11, 14, \ldots$
b. $4, 10, 16, 22, \ldots$
c. $20, 17, 14, 11, \ldots$
d. $7, 12, 17, 22, \ldots$

Question 4

Question 4 – Is it a term? *(Section A · Skills practice)*

To check whether a number is in a sequence, set the nth term equal to the number and solve. It is in the sequence only if n is a positive whole number.

a. Is 100 a term of $3n + 1$?
b. Is 150 a term of $3n + 1$?
c. Is 63 a term of $5n - 2$?
d. Is 80 a term of $5n - 2$?

Question 5

Question 5 – Geometric sequences *(Section A · Skills practice)*

In a geometric sequence you multiply by the same number, the common ratio r, each time.

a. $3, 6, 12, 24, \ldots$ Find r and the next two terms.
b. $81, 27, 9, 3, \ldots$ Find r and the next two terms.
c. The sequence $5, 10, 20, \ldots$ has nth term $5 \times 2^{n-1}$. Find the 8th term.
d. Is $3, 6, 9, 12$ geometric? Explain.

Question 6

Question 6 – Special sequences *(Section A · Skills practice)*

Some sequences have famous names. The square numbers are $1, 4, 9, 16, \ldots$, the cube numbers are $1, 8, 27, 64, \ldots$ and the triangular numbers are $1, 3, 6, 10, \ldots$ In a Fibonacci-type sequence each term is the sum of the two terms before it.

a. Write the next three triangular numbers.
b. Find the 10th square number and the 6th cube number.
c. A Fibonacci-type sequence starts $2, 3, \ldots$ Write the next three terms.
d. Powers of 2 start $2, 4, 8, 16, \ldots$ Find the 10th term.

Question 7

Question 7 – Matchstick squares *(Section B · Real-life practice)*

Tom makes a row of squares using matchsticks. One square uses 4 matchsticks, two squares use 7, and three squares use 10.



a. Write down the number of matchsticks for 4 squares and 5 squares.
b. Find the nth term for the number of matchsticks for n squares.
c. How many matchsticks are needed for 20 squares?
d. Tom has 151 matchsticks. How many squares can he make?

Question 8

Question 8 – A savings plan *(Section B · Real-life practice)*

Ruth has £15 in her money box and adds £8 at the end of every week. Let the amount after n weeks be A_n.

Week n	1	2	3	4
Amount A_n (£)	23	31	39	47

a. Find the nth term for the amount.
b. How much will she have after 12 weeks?
c. After how many weeks will she first have at least £200?
d. Her brother has £60 and saves £5 a week. When does Ruth first have more than her brother?

Question 9

Question 9 – The theatre seats *(Section B · Real-life practice)*

A school theatre has 12 seats in the first row. Each row behind has 2 more seats than the row in front.

a. How many seats are in rows 2, 3 and 4?
b. Find the nth term for the number of seats in row n.
c. How many seats are in row 15?
d. There are 8 rows altogether. Find the total number of seats.

Question 10

Question 10 – The bouncing ball *(Section B · Real-life practice)*

A ball is dropped from 160 cm. Each time it bounces it reaches half the height of the previous bounce.

Bounce	1	2	3	4
Height (cm)	80	40	20	10

a. What type of sequence is this? State the common ratio.
b. Find the height of the 6th bounce.
c. Write an expression for the height of the nth bounce.
d. After which bounce is the height first below 1 cm?

Question 11

Question 11 – Tiles round a pond *(Section B · Real-life practice)*

Mrs Evans is building a square pond from 1 slab, then 2 by 2 slabs, then 3 by 3 slabs. She surrounds each pond with a border of black tiles one tile wide, as in the table.

Pond size	1 by 1	2 by 2	3 by 3
Black tiles	8	12	16

a. Find the nth term for the number of black tiles around an n by n pond.
b. How many black tiles are needed for a 10 by 10 pond?
c. A pond needs 84 black tiles. What size is it?
d. Explain the formula by looking at the corners and the sides of the border.

Question 12

Question 12 – Rabbit numbers *(Section B · Real-life practice)*

A pair of rabbits is bred. In the Fibonacci model, the number of pairs each month is the sum of the numbers in the previous two months. The first two months have 1 pair each.

Month	1	2	3	4	5	6
Pairs	1	1	2	3	5	8

a. Find the number of pairs in months 7, 8 and 9.
b. Find the number in month 10.
c. Does this sequence have a constant difference or a constant ratio? Explain.
d. The ratios of consecutive terms get close to 1.6. Check this for months 9 and 10.

Question 13

Question 13 – Stacking tins in the shop *(Section B · Real-life practice)*

A supermarket stacks tins in a triangle: 1 tin in the top row, 2 in the second, 3 in the third, and so on. The numbers of tins in the whole stack are the triangular numbers $1, 3, 6, 10, \ldots$

a. Write the number of tins in a stack with 4 rows and with 8 rows.
b. The formula for the number in a stack with n rows is $\dfrac{n(n+1)}{2}$. Check it for $n = 4$.
c. How many tins are in a 10-row stack?
d. A stack has 45 tins. How many rows does it have?

Question 14

Question 14 – The training plan *(Section B · Real-life practice)*

Hamza starts a running plan. In week 1 he runs 2 km. Every week after that he adds 0.5 km to the length of his weekly long run.

Week	1	2	3	4
Distance (km)	2	2.5	3	3.5

a. Find the nth term for the distance in week n.
b. How far does he run in week 10?
c. In which week will he first run 10 km or more?
d. After the plan ends he runs 5 km a week. Is it likely that the formula keeps working for ever? Explain.

Question 15

Question 15 – The sequence table *(Section C · Problem solving)*

A teacher gives four sequences. Complete the table.

Sequence	nth term	10th term	50th term
$4, 7, 10, 13, \ldots$	?	?	?
$2, 9, 16, 23, \ldots$	?	?	?
$30, 26, 22, 18, \ldots$	?	?	?
$1, 4, 9, 16, \ldots$	?	?	?

a. Complete the table.
b. Which sequence is not arithmetic? How can you tell?
c. Which sequence has a negative 50th term?
d. For which value of n does the third sequence first become negative?

Question 16

Question 16 – Two sequences *(Section C · Problem solving)*

Sequence A has nth term $3n + 2$ and sequence B has nth term $5n - 3$.

n	1	2	3	4	5	6
A	5	8	11	14	17	20
B	2	7	12	17	22	27

a. Which number appears in both sequences in the table?
b. Show that 32 is a term of both sequences and find n for each.
c. Show that there is no value of n at which A and B have the same term in the same position.
d. Which sequence grows faster, and why?

Question 17

Question 17 – A quadratic sequence *(Section C · Problem solving)*

Look at the sequence $3, 8, 15, 24, 35, \ldots$

n	1	2	3	4	5

n	1	2	3	4	5
Term	3	8	15	24	35
First difference		5	7	9	11
Second difference			2	2	2

a. What is special about the second differences?
b. The nth term has the form $n^2 + bn$. Use the first term to find b.
c. Check your formula with $n = 4$.
d. Find the 10th term.
e. Is 99 a term of the sequence?

Question 18

Question 18 – A rule from two terms *(Section C · Problem solving)*

An arithmetic sequence has 4th term 17 and 9th term 42.

a. Find the common difference.
b. Find the first term and the nth term.
c. Find the 20th term.
d. Is 497 a term of the sequence? Is 500?

Question 19

Question 19 – The chain letter *(Section C · Problem solving)*

A chain letter is started by one person on day 1. Each person who receives it sends it to 3 new people the next day. The numbers who receive it each day are $1, 3, 9, 27, \ldots$

Day	1	2	3	4	5
People	1	3	9	27	81

a. Find the common ratio and the nth term.
b. How many receive the letter on day 8?
c. The population of a town is 100 000. On which day does the number who receive the letter first exceed this?
d. Explain why the chain cannot really continue for ever.

Question 20

Question 20 – Challenge: triangular numbers *(Section D · Challenge)*

The triangular numbers are $T_n = \dfrac{n(n+1)}{2}$: $1, 3, 6, 10, 15, \ldots$ and the square numbers are n^2.

a. Write down T_1 to T_5.

b. Show that $T_4 + T_5 = 25$.
c. Show by algebra that $T_n + T_{n+1} = (n+1)^2$.
d. Use the result to find $T_{19} + T_{20}$ and check by using the formula.
e. Find the first triangular number greater than 200.