Answers - Sequences and nth term

Year: Year 9

Answer to Question 1

a. Add 4: **19, 23** The difference between consecutive terms is always 4, so this is an arithmetic sequence.
b. Subtract 10: **60, 50** Each term is 10 less than the one before, so the sequence is decreasing.
c. Multiply by 3: **162, 486** Each term is three times the one before, so this is a geometric sequence with ratio 3.
d. Square numbers $1^2, 2^2, 3^2, 4^2$: **25, 36** The differences are 3, 5, 7, so they are not constant; the terms are the square numbers.

Answer to Question 2

a. $5, 8, 11, 14$. 10th: $3 \times 10 + 2 =$ **32** Substitute $n = 1$, 2, 3 and 4 in turn. The terms go up by 3 because the coefficient of n is 3.
b. $2, 5, 10, 17$. 5th: $25 + 1 =$ **26** Square n first and then add 1. The differences are 3, 5, 7, so the sequence is not arithmetic.
c. $46, 42, 38$. 12th: $50 - 48 =$ **2** Substitute $n = 1$, 2, 3: the terms go down by 4 each time.
d. $2, 4, 8, 16, 32$ Each term is double the one before; these are the powers of 2.

Answer to Question 3

a. $d = 3$, $a - d = 2$: $3n + 2$ The common difference is 3, and the term before the first would be $5 - 3 = 2$. Check: $3 \times 4 + 2 = 14$.
b. $d = 6$, $a - d = -2$: $6n - 2$ The common difference is 6, and $4 - 6 = -2$. Check: $6 \times 4 - 2 = 22$.
c. $d = -3$, $a - d = 23$: $-3n + 23$ (or $23 - 3n$) The terms go down by 3, so $d = -3$, and $20 - (-3) = 23$. Check: $23 - 12 = 11$.
d. $d = 5$, $a - d = 2$: $5n + 2$ The common difference is 5, and $7 - 5 = 2$. Check: $5 \times 4 + 2 = 22$.

Answer to Question 4

a. $3n + 1 = 100$, $n = 33$. **Yes**, it is the 33rd term. Subtract 1 and divide by 3. The answer is a whole number, so 100 is in the sequence.
b. $3n + 1 = 150$, $n = 49.67$. **No.** The solution is not a whole number, so 150 lies between the 49th and 50th terms.

c. $5n - 2 = 63$, $n = 13$. **Yes.** Add 2 and divide by 5. The answer is a whole number, so 63 is the 13th term.
d. $5n - 2 = 80$, $n = 16.4$. **No.** The solution is not a whole number, so 80 is not in the sequence.

Answer to Question 5

a. $r = 2$: **48, 96** Divide any term by the one before: $6 \div 3 = 2$. Then keep multiplying by 2.
b. $r = \frac{1}{3}$: **1,** $\frac{1}{3}$ Each term is one third of the one before, so keep dividing by 3.
c. $5 \times 2^7 = 5 \times 128 =$ **640** Substitute $n = 8$: the power is $8 - 1 = 7$, and $2^7 = 128$.
d. No. It adds 3 each time, so it is arithmetic, not geometric. The ratios $6 \div 3 = 2$ and $9 \div 6 = 1.5$ are different, so there is no common ratio.

Answer to Question 6

a. Add 5, 6, 7: **15, 21, 28** The triangular numbers add 2, 3, 4, ... each time, so the next additions are 5, 6 and 7.
b. $10^2 =$ **100** and $6^3 =$ **216** The 10th square number is 10×10 and the 6th cube number is $6 \times 6 \times 6$.
c. $2 + 3 = 5$; $3 + 5 = 8$; $5 + 8 = 13$: **5, 8, 13** Add the two previous terms each time to continue the sequence.
d. $2^{10} =$ **1 024** The 10th power of 2 is $2 \times 2 \times \cdots \times 2$ with ten 2s.

Answer to Question 7

a. 4 squares: **13**; 5 squares: **16** (add 3 each time). Each new square shares one side with the previous square, so it needs only 3 more matchsticks.
b. $d = 3$, so $3n + 1$. Check: $n = 1$ gives 4, $n = 2$ gives 7. The common difference is 3 and $4 - 3 = 1$, so the nth term is $3n + 1$.
c. $3 \times 20 + 1 =$ **61** Substitute $n = 20$ into the formula.
d. $3n + 1 = 151$, so $3n = 150$ and $n =$ **50 squares**. Set the formula equal to 151 and solve the equation. The answer is a whole number, so all 151 matchsticks are used.

Answer to Question 8

a. $d = 8$, so $A_n = 8n + 15$ The amounts go up by 8 each week, and the term before the first would be $23 - 8 = 15$, the starting amount.
b. $8 \times 12 + 15 =$ **£111** Substitute $n = 12$ into the formula.
c. $8n + 15 \geq 200$, $n \geq 23.1$, so **24 weeks**. Solve the inequality and round up,

because after 23 weeks she has only £199.

d. Brother: $5n + 60$. Ruth has more when $8n + 15 > 5n + 60$, $3n > 45$, $n > 15$: first in **week 16**. Write her brother's amount as a formula and solve the inequality. In week 15 they both have £135.

Answer to Question 9

a. Row 2: **14**, row 3: **16**, row 4: **18**. Add 2 seats for each row further back.
b. $d = 2$, so $2n + 10$. The common difference is 2, and the number before the first term is $12 - 2 = 10$.
c. $2 \times 15 + 10 =$ **40 seats** Substitute $n = 15$ into the formula.
d. Row 8 has $2 \times 8 + 10 = 26$. The rows form $12, 14, \ldots, 26$: 8 terms with average 19, so total $= 8 \times 19 =$ **152 seats**. Pair the first and last rows: $12 + 26 = 38$, and there are 4 such pairs, so $4 \times 38 = 152$ as well.

Answer to Question 10

a. **Geometric**, common ratio $\frac{1}{2}$. Each height is half of the one before, so the sequence is geometric with ratio $\frac{1}{2}$.
b. $160 \times (\frac{1}{2})^6 = 160 \div 64 =$ **2.5 cm** Keep halving: 10, 5, then 2.5 cm.
c. $160 \times \left(\frac{1}{2}\right)^n$ Start from 160 and multiply by $\frac{1}{2}$ once for each bounce.
d. Bounce 7: 1.25; bounce 8: 0.625. So after the **8th bounce**. Keep halving and compare each height with 1 cm.

Answer to Question 11

a. $d = 4$, so $4n + 4$ The number of black tiles goes up by 4 each time, and $8 - 4 = 4$.
b. $4 \times 10 + 4 =$ **44 tiles** Substitute $n = 10$ into the formula.
c. $4n + 4 = 84$, so $n =$ **20**, a 20 by 20 pond. Set the formula equal to 84 and solve. Check: $4 \times 20 + 4 = 84$.
d. There are 4 corner tiles. Each side needs n tiles, and there are 4 sides, so $4n + 4$. The border is made of four sides of n tiles plus one tile in each of the four corners.

Answer to Question 12

a. Month 7: $5 + 8 = 13$; month 8: 21; month 9: **34**. Add the two previous numbers each time: $5 + 8$, $8 + 13$, $13 + 21$.
b. Month 10: $21 + 34 =$ **55**. Add the numbers for months 8 and 9.
c. Neither. The differences are 0, 1, 1, 2, 3, 5, so they change; the ratios 1, 2, 1.5, 1.67, 1.6 also change. In an arithmetic sequence the differences are constant,

and in a geometric one the ratios are constant. Neither happens here.

d. $55 \div 34 = 1.618\ldots \approx 1.6$. Divide the month 10 number by the month 9 number.

Answer to Question 13

a. 4 rows: $1 + 2 + 3 + 4 =$ **10**. 8 rows: $1 + 2 + \cdots + 8 =$ **36**. Add the number of tins in each row: $1 + 2 + 3 + 4$, and so on up to 8 rows.

b. $\dfrac{4 \times 5}{2} = 10$. Correct. Substituting $n = 4$ gives the same answer as adding the rows.

c. $\dfrac{10 \times 11}{2} =$ **55** Substitute $n = 10$ into the formula.

d. $\dfrac{n(n+1)}{2} = 45$ gives $n(n+1) = 90$, so $n =$ **9** rows ($9 \times 10 = 90$). Find two consecutive numbers that multiply to 90.

Answer to Question 14

a. $d = 0.5$, $a - d = 1.5$: $0.5n + 1.5$ The distance goes up by 0.5 km each week, and $2 - 0.5 = 1.5$.

b. $0.5 \times 10 + 1.5 =$ **6.5 km** Substitute $n = 10$ into the formula.

c. $0.5n + 1.5 \geq 10$, $n \geq 17$, so **week 17**. Solve the inequality: $0.5n \geq 8.5$, so $n \geq 17$.

d. No. The formula would give impossible distances, such as 50 km a week, so the model only works for a limited time. Real runners need rest and cannot keep increasing their distance for ever.

Answer to Question 15

a. $3n + 1$: 31, 151. $7n - 5$: 65, 345. $34 - 4n$: -6, -166. n^2: 100, 2500. Find the common difference and the zero term for each arithmetic sequence, then substitute $n = 10$ and $n = 50$.

b. The last: differences are 3, 5, 7 (not constant), so it is not arithmetic. Look at the first differences: for an arithmetic sequence they must all be equal.

c. The third sequence: $34 - 200 = -166$. Only the decreasing sequence will go below zero.

d. $34 - 4n$ gives $n > 8.5$, so first negative at $n = 9$ (the term is -2). Solve the inequality and take the next whole number above 8.5.

Answer to Question 16

a. **17** appears in both (A has it at $n = 5$, B at $n = 4$). Look for a number that appears in both rows of the table.

b. A: $3n + 2 = 32$, $n = 10$. B: $5n - 3 = 32$, $n = 7$. Solve each equation; both answers are whole numbers, so 32 is in both sequences.
c. $3n + 2 = 5n - 3$ gives $5 = 2n$, so $n = 2.5$, not a whole number. For A and B to be equal in the same position, n would have to be 2.5, which is not a position.
d. **B** grows faster because its common difference (5) is greater than A's (3). The common difference is the coefficient of n, and 5 is bigger than 3.

Answer to Question 17

a. The second differences are all 2, so the sequence is **quadratic** with n^2 as the first part. Constant second differences show that the nth term contains n^2; the coefficient of n^2 is half the second difference, which is 1.
b. $1 + b = 3$, so $b = 2$: nth term $= n^2 + 2n$. Substitute $n = 1$ into $n^2 + bn$ and set it equal to the first term, 3.
c. $16 + 8 = 24$. Correct. Substitute $n = 4$: the answer matches the 4th term in the table.
d. $100 + 20 =$ **120** Substitute $n = 10$ into $n^2 + 2n$.
e. $n^2 + 2n = 99$ gives $n^2 + 2n + 1 = 100$, so $n + 1 = 10$, $n = 9$: **yes**, it is the 9th term. Adding 1 to both sides gives $(n + 1)^2 = 100$, so $n + 1 = 10$.

Answer to Question 18

a. Between the 4th and 9th terms there are 5 steps: $42 - 17 = 25$, so $d = 25 \div 5 =$ **5**. Going from the 4th term to the 9th term means adding the common difference 5 times.
b. 4th term $= 17$, so $a = 17 - 3 \times 5 =$ **2**. nth term $= 5n - 3$. Go back 3 steps from the 4th term to find the first term, then use $dn + (a - d)$.
c. $5 \times 20 - 3 =$ **97** Substitute $n = 20$ into $5n - 3$.
d. $5n - 3 = 497$ gives $n = 100$: **yes**. $5n - 3 = 500$ gives $n = 100.6$: **no**. A number is a term only if the solution n is a positive whole number.

Answer to Question 19

a. $r = 3$, nth term $= 3^{n-1}$. Each day three times as many people receive the letter, so the ratio is 3.
b. $3^7 =$ **2 187** Substitute $n = 8$ into 3^{n-1}.
c. Day 11: $3^{10} = 59\,049$. Day 12: $3^{11} = 177\,147$. So on **day 12**. Use the power key to find the first power of 3 bigger than 100 000.
d. The number of people is finite, so eventually everyone has received it and the numbers must stop growing. Geometric growth soon gives numbers bigger than the population of the whole world.

Answer to Question 20

a. $1, 3, 6, 10, 15$ Substitute $n = 1$ to 5 into the formula, or keep adding 2, 3, 4, 5.

b. $T_4 + T_5 = 10 + 15 = 25 = 5^2$ Add the 4th and 5th triangular numbers; the total is a square number.

c. $\frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} = \frac{(n+1)(n+n+2)}{2} = \frac{(n+1)(2n+2)}{2} = (n+1)^2$. Take out the common factor $(n+1)$ and simplify the second bracket.

d. $T_{19} + T_{20} = 20^2 = 400$. Check: $190 + 210 = 400$. Use $T_{19} = \frac{19 \times 20}{2} = 190$ and $T_{20} = \frac{20 \times 21}{2} = 210$.

e. $T_{20} = 210$ is the first above 200 (since $T_{19} = 190$). $T_{19} = 190$ is below 200 and $T_{20} = 210$ is above it.