Lesson - Real-life graphs

Year: Year 9

This lesson covers distance-time graphs, conversion graphs, linear graphs in context, piece-wise graphs, exponential and reciprocal graphs, and reading approximate solutions.

Starter activity

Walk across the classroom at a steady pace while a friend times you every metre. Plot distance against time. Then walk faster, stop for a few seconds, and walk again. How does your graph change? Which part is steepest, and which part is flat? What would a graph look like for someone who walked back to the start?

Graphs that tell a story

Note: reading a real-life graph

In real-life graphs the axes are labelled with quantities such as distance, time, cost or population, and every part of the graph has a meaning. Read values, work out gradients, describe what each section shows and always give units.

Distance-time graphs

Key fact: distance-time graphs

Time goes along the horizontal axis and **distance from the start** up the vertical axis.

- A straight sloping line means constant speed.
- A **horizontal** line means the object is stationary.
- A steeper line means a faster speed.
- A line going back down means the object is returning towards the start.

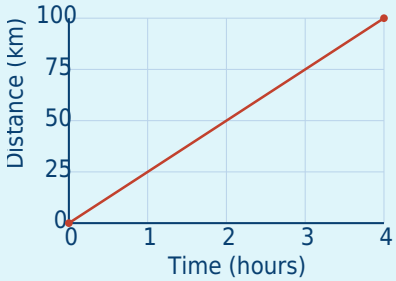
Rule: gradient is speed

The **gradient** of a distance-time graph is the speed: $\text{speed} = \dfrac{\text{distance}}{\text{time}}$, which is also the change in y divided by the change in x.

Worked example 1: speed from a gradient

A line goes from $(0, 0)$ to $(4, 100)$, with time in hours and distance in km. Find the speed.

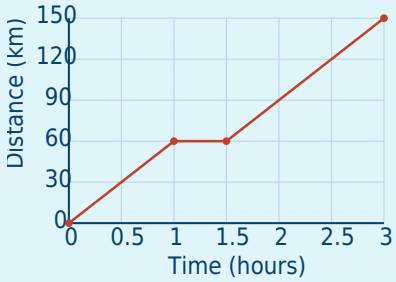
Gradient $= \dfrac{100}{4} = 25$, so the speed is 25 km/h.



Worked example 2: average speed with a rest

A cyclist travels 60 km in the first hour, rests for half an hour, then travels a further 90 km, reaching the end at 3 hours. Find the average speed.

Total distance is 150 km in 3 hours, so the average speed is $\dfrac{150}{3} = 50$ km/h. The rest counts in the total time.



Conversion graphs

Method: using a conversion graph

1. Find the value on one axis.
2. Go across or up to the line.
3. Read the other axis.

A conversion graph such as miles to kilometres is a straight line through the origin: since 5 miles is about 8 km, 20 miles is $20 \times \dfrac{8}{5} = 32$ km. For temperatures the line does not pass through the origin, because 0°C is 32°F.

Linear graphs in context

Rule: meaning of m and c

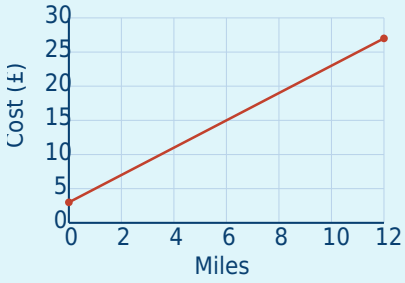
A straight-line graph has equation $y = mx + c$. In a real-life setting the **gradient** m is the rate of change (the cost per mile, per minute or per unit) and the **y-intercept** c is

the starting value or fixed charge.

Worked example 3: taxi fares

A taxi charges £3 plus £2 per mile, so $C = 2m + 3$. Find the cost of 7 miles and the distance for a £25 fare.

For 7 miles, $C = 14 + 3 = £17$. For £25: $2m + 3 = 25$, $2m = 22$, $m = 11$ miles.



Piece-wise linear graphs

Definition: piece-wise linear graph

A **piece-wise** graph is made of several straight sections with different gradients, and each section needs its own gradient.

Worked example 4: car park charges

The first hour in a car park costs £2 and each extra hour costs £1.50. Find the cost for 4 hours.

The cost is $2 + 3 \times 1.50 = £6.50$.

Exponential graphs

Definition: exponential graph

An **exponential** graph, such as $N = 100 \times 2^t$, describes something multiplied by the same factor in equal time periods, such as bacteria doubling every hour. The curve gets steeper and steeper.

t (hours)	0	1	2	3	4
N	100	200	400	800	1600

Worked example 5: exponential decay

A car worth £12 000 loses 20% of its value each year. Find its value after 1 and 2 years.

After 1 year: $12\,000 \times 0.8 = £9600$. After 2 years: $9600 \times 0.8 = £7680$. In exponential decay the curve falls steeply at first and then flattens.

Reciprocal graphs

Definition: reciprocal graph

A **reciprocal** graph has the form $y = \frac{k}{x}$. It is a curve in two branches that never touches the axes. In context it models inverse relationships, such as the time t hours to travel 120 km at speed v: $t = \frac{120}{v}$. At 40 km/h the time is 3 hours; at 60 km/h it is 2 hours.

Reading approximate solutions

Note: reading values

Draw a line from the given value on one axis to the curve, then straight down or across to the other axis. Because graphs are drawn by hand, answers are approximate, so give a sensible degree of accuracy and say the answer is an estimate.

Watch out: common mistakes

- Reading the scale wrongly: always work out what each small square is worth.
- Saying a horizontal line on a distance-time graph means constant speed. It means the object is not moving.
- Forgetting units in the gradient (km/h, not just 25).
- Forgetting to include rest time in an average speed calculation.

Worked example 6: average speed from a graph

A runner covers 6 km in 36 minutes, rests for 4 minutes and then covers 4 km in 24 minutes. Find the average speed for the whole run, in km/h.

Total distance $= 6 + 4 = 10$ km. Total time $= 36 + 4 + 24 = 64$ minutes $= \frac{64}{60}$ hours.

Average speed $= 10 \div \frac{64}{60} = 9.375$ km/h, so about 9.4 km/h. The rest is included in the time because average speed uses the whole time.

Worked example 7: comparing two tariffs

Plan A costs £5 plus 10p per minute. Plan B costs 20p per minute. When are

they equal?

A: $y = 5 + 0.1x$. B: $y = 0.2x$. Set equal: $5 + 0.1x = 0.2x$, so $5 = 0.1x$ and $x = 50$ minutes. For under 50 minutes B is cheaper, and for more than 50 minutes A is cheaper.

Worked example 8: reading an exponential graph

A population is $P = 1000 \times 1.1^t$. Find the population after 3 years and 7 years.

After 3 years: $1000 \times 1.331 = 1331$. After 7 years: $1000 \times 1.9487 = 1949$ (to the nearest whole number). The increases are 331 in the first 3 years but 618 in the next 4 years, so the growth gets bigger each year.

Worked example 9: a reciprocal relation

The time to paint a fence is $t = \dfrac{18}{n}$ hours with n painters. Find t for 6 painters and the number needed for 1.5 hours.

$t = 18 \div 6 = 3$ hours. For 1.5 hours: $18 \div 1.5 = 12$ painters. The product $n \times t = 18$ is constant, which is the hallmark of a reciprocal relation.

Note: label and scale

Always give a graph a title, label the axes with quantities and units, and choose a scale that fills the page. When you read a value from a graph, draw lines to the axes and write down the units. Remember that the speed for a section is the gradient, so a steeper section means faster travel and a horizontal section means the object is not moving.

Watch out: graphs that look like journeys

- A distance-time graph is not a picture of the road: a steep line is fast, not a hill.
- A line going *down* on a distance-time graph means coming back towards the start, not slowing down.
- Average speed is total distance divided by total time, including the stops. It is not the mean of the speeds.

Did you know?

The word exponential describes growth that gets faster as the amount gets larger. This is why early warnings about the spread of infections or the growth of savings with compound interest use graphs: a small difference in the growth rate leads to a huge difference after only a few years.

Describing and sketching graphs

Method: describing a journey from a graph

1. Split the graph into sections where the line is straight.
2. For each section, say whether the object is moving away, coming back or not moving at all (a horizontal line).
3. Work out the speed in each section using the gradient: distance travelled divided by time taken.
4. Compare the sections: the steeper the line, the faster the movement.
5. Write your description in full sentences with units, for example "From 10:00 to 11:00 the cyclist travelled 15 km away from home at 15 km/h."

Worked example 10: sketching a filling graph

Water is poured at a steady rate into a vase that is narrow at the bottom and wide at the top. Sketch the graph of the depth of water against time.

At the start the vase is narrow, so a small amount of water makes the level rise quickly and the graph is steep. As the vase gets wider, the same amount of water spreads out over a larger area, so the level rises more slowly and the graph becomes less steep. The sketch is a curve that starts steep and gradually flattens. If the vase had straight vertical sides, the depth would rise at a constant rate and the graph would be a straight line through the origin.

Note: units on the axes

The units of a gradient come from the units on the two axes. On a distance-time graph in kilometres and hours, the gradient is in kilometres per hour. On a graph of cost in pounds against units of electricity, the gradient is in pounds per unit. Writing the units helps you to explain what the gradient means in real life.

Otto's tip

Before answering, say out loud what the axes represent and what a steeper line means. It saves marks. Hoot hoot!