# Answers - Real-life graphs

Year: Year 9

## Answer to Question 1

a. Gradient $= 40 \div 1 =$ **40 km/h** In the first hour the coach travels 40 km, so the speed is the gradient of that section.
b. The line is flat between 1 and 1.5 hours, so the coach **stopped for 0.5 hours (30 minutes)**. A horizontal line means the distance from the start does not change, so the coach is not moving.
c. Distance $= 130 - 40 = 90$ km in $1.5$ hours: $90 \div 1.5 =$ **60 km/h** The coach travels 90 km in 1.5 hours, so the line is steeper than in the first section.
d. $130 \div 3 = 43.3$, so **43.3 km/h** Average speed = total distance ÷ total time, including the stop. The coach averaged 43.3 km/h.

## Answer to Question 2

a. $8 \div 5 = 1.6$: **1 mile is 1.6 km**. The gradient is the change in kilometres divided by the change in miles.
b. $30 \times 1.6 =$ **48 km** Multiply the number of miles by 1.6.
c. $100 \div 1.6 =$ **62.5 miles** To change kilometres to miles, divide by 1.6.
d. $1.8 \times 25 + 32 = 45 + 32 =$ **77°F** Multiply by 1.8 first and then add 32, following the order of operations.

## Answer to Question 3

a. 30 is the **hourly rate** (the gradient). 40 is the **call-out fee** (the $y$-intercept). In $C = mh + c$, the gradient is the cost per hour and the intercept is the cost before any work is done.
b. $30 \times 2.5 + 40 =$ **£115** Substitute $h = 2.5$ into the formula.
c. $175 - 40 = 135$, $135 \div 30 =$ **4.5 hours** Subtract the call-out fee first, then divide by the hourly rate. Check: $30 \times 4.5 + 40 = 175$.
d. At $(0, 40)$, where $h = 0$. The graph crosses the vertical axis when $h = 0$, which gives the call-out fee.

## Answer to Question 4

a. $15 \div 1 =$ **15 km/h** Speed = distance ÷ time for the first hour.
b. $(55 - 15) \div 2 = 40 \div 2 =$ **20 km/h** In part 2 the distance increases by 40 km in 2 hours.
c. The distance does not change, so the cyclist **stopped** (speed 0). The distance

stays at 55 km for an hour, so the graph would be horizontal.

d. **Part 2** (20 km/h): it has the steepest line. The fastest part has the greatest speed, which is the steepest line on a distance-time graph.

## Answer to Question 5

a. $50, 100, 200, 400, 800, 1600$ Multiply by 2 for each extra hour.
b. $N = 50 \times 2^t$ The starting number is 50, and 2 is multiplied once for every hour.
c. $50 \times 2^{10} = 50 \times 1024 =$ **51 200** Substitute $t = 10$: $2^{10} = 1024$.
d. The graph is a **curve** that gets steeper and steeper, because the amount added each hour grows. In a linear graph the same amount is added each hour; here the amount is multiplied, so the increase gets bigger.

## Answer to Question 6

a. $12, 6, 4, 3, 2, 1$ Divide 12 by each value of $x$; the product $xy$ is always 12.
b. $12 \div 0.5 =$ **24** Dividing by 0.5 is the same as multiplying by 2.
c. $12 \div y = 12 \div 0.5 =$ **24** Since $xy = 12$, $x = 12 \div y$.
d. Division by zero is not possible, so the graph never touches the $y$-axis. As $x$ gets closer to 0, $y$ gets bigger and bigger, so the curve never reaches the axis.

## Answer to Question 7

a. $12 \div 1 =$ **12 km/h** She cycles 12 km in 1 hour.
b. She arrives at 11:00 and rests for 30 minutes, so she leaves at **11:30**. She arrives at 11:00 and rests for half an hour.
c. $40$ min $= \frac{2}{3}$ hour, $12 \div \frac{2}{3} =$ **18 km/h** 40 minutes is two thirds of an hour, so divide 12 by $\frac{2}{3}$.
d. Total distance $24$ km. Total time $1 + 0.5 + \frac{2}{3} = 2\frac{1}{6}$ h, so $24 \div 2.1667 =$ **11.1 km/h**. The total distance is there and back, and the total time includes the rest.

## Answer to Question 8

a. $10 + 2 \times (12 - 5) = 10 + 14 =$ **£24** The first 5 GB cost £10, and 7 extra GB cost £2 each.
b. For $x \leq 5$: $y = 10$. For $x > 5$: $y = 10 + 2(x - 5) = 2x$. Up to 5 GB the cost is fixed, so the graph is horizontal; after that it rises with gradient 2.
c. $10 + 2(x - 5) = 24$, so $x - 5 = 7$ and $x =$ **12 GB**. Set the formula equal to 24 and solve. Check: $2 \times 12 = 24$.
d. Competitor: $12 \times 1.5 = £18$, which is **cheaper** than £24. Multiply the competitor's rate by 12 and compare with £24. The competitor is £6 cheaper.

## Answer to Question 9

a. Within 1 hour: **£1.50** 45 minutes is less than 1 hour, so the first price band applies.
b. Within 4 hours: **£4.00** 3 hours 15 minutes is more than 2 hours but less than 4 hours.
c. 4 hours cost £4.00, then 3 extra hours cost £3.00: **£7.00** After 4 hours each extra hour costs £1, and 7 hours is 3 hours more than 4.
d. $4 + 5 = 9$, so **9 hours**. Any longer is still £9.00, since that is the daily maximum. Each hour after 4 hours adds £1, so £9 is reached after 5 extra hours.

## Answer to Question 10

a. £50 gives $50 \times 1.2 =$ **€60**. Pounds and euros are in direct proportion, so the graph is a straight line through the origin with gradient 1.2.
b. $90 \div 1.2 =$ **£75** To change euros into pounds, divide by 1.2.
c. $E = 1.2P$ The gradient of the line is 1.2 and the line passes through the origin.
d. $36 \div 1.2 =$ **£30** Divide the euros by 1.2 to change them back into pounds.

## Answer to Question 11

a. $40 + 15 \times 20 =$ **340 litres** The tank gains 15 litres per minute for 20 minutes, added to the 40 litres at the start.
b. $V = 15t + 40$ The starting volume is the intercept and the filling rate is the gradient.
c. $340 \div 25 =$ **13.6 minutes** The tank loses 25 litres every minute, so divide the 340 litres by 25.
d. $20 + 13.6 =$ **33.6 minutes** Add the 20 minutes of filling to the 13.6 minutes of draining.

## Answer to Question 12

a. Hour 3: **1 600**; hour 4: **3 200**; hour 5: **6 400**; hour 6: **12 800**. Multiply by 2 for each extra hour.
b. $N = 200 \times 2^t$ The starting number is 200, multiplied by 2 once for every hour.
c. At hour 5 it is 6 400 and at hour 6 it is 12 800, so it first exceeds 10 000 after **6 hours**. Look at the table: 6 400 is still below 10 000, but 12 800 is above it.
d. The dish has limited food and space, so growth cannot continue to double. An exponential model only works for a limited time in real life.

## Answer to Question 13

a. $6, 5, 4, 3, 2$ hours Divide 240 by each speed; for example $240 \div 48 = 5$.
b. A **curve** that falls quickly at first and then flattens (reciprocal graph). Speed × time is always 240, so this is a reciprocal (inverse proportion) graph.
c. $240 \div 2.5 =$ **96 km/h** Rearrange the formula: $s = 240 \div t$.
d. $240 \div 90 = 2.67$ h = **2 hours 40 minutes**. The decimal 0.67 of an hour is two thirds of 60 minutes, which is 40 minutes.

## Answer to Question 14

a. $60, 40, 30, 24, 20, 15$ Divide £120 by each number of pupils.
b. It **decreases**, but never reaches zero. The total cost is shared between more people, so each share is smaller.
c. $120 \div 10 =$ **12 pupils** Rearrange the formula: $n = 120 \div 10$.
d. $15 + 2.50 =$ **£17.50** Add the snack price to the minibus share for 8 pupils.

## Answer to Question 15

a. $d = 10t$ Distance = speed × time, and Tom starts at $t = 0$.
b. $d = 20(t - 0.5) = 20t - 10$ Anna starts half an hour later, so she has been cycling for $t - 0.5$ hours.
c. They meet at $t = 1$ hour, at 10 km: **11:00 and 10 km from the start**. The two lines cross where both cyclists are at the same place at the same time.
d. $10t = 20t - 10$ gives $10t = 10$, so $t = 1$ and $d = 10$. Setting the two distances equal and solving gives the same point as the graph.
e. Tom takes $25 \div 10 = 2.5$ h and Anna takes $0.5 + 25 \div 20 = 1.75$ h, so Tom arrives **0.75 h (45 min)** after Anna. Work out each finishing time from the start at 10:00 and subtract.

## Answer to Question 16

a. $(50 - 3) \times 1.20 = 47 \times 1.2 =$ **€56.40** Take the £3 fee off first, then multiply the remaining pounds by 1.20.
b. Bureau: $E = 1.2(P - 3) = 1.2P - 3.6$. Bank: $E = 1.18P$. The bureau formula subtracts the fee before converting; the bank formula is a straight line through the origin.
c. $1.2 \times 177 = 212.4$ and $1.18 \times 180 = 212.4$. Both formulas give €212.40, so the lines cross at $P = 180$.
d. £500: bureau $1.2 \times 497 = 596.4$; bank $590$, so **bureau** is better. £60: bureau $68.4$; bank $70.8$, so **bank** is better. Above £180 the bureau's better rate outweighs its fee; below £180 the fee makes the bank better.

## Answer to Question 17

a. $8 + 20 + 0.15 \times 100 = 8 + 20 + 15 =$ **£43** The first 100 units cost 20p each and the next 100 cost 15p each, plus the standing charge.
b. $28 + 0.15 \times 200 =$ **£58** The bill for 100 units is £28; add 200 more units at 15p.
c. $28 + 0.15 \times 150 =$ **£50.50** Add 150 units at 15p to the £28 for the first 100 units.
d. The price per unit changes from 20p to 15p, so the gradient is smaller after 100 units. The gradient of a cost graph is the price per unit, and the price falls after 100 units.
e. £58 corresponds to **300 units**. Subtract £28 and divide by 0.15 to find the units above 100: $30 \div 0.15 = 200$.

## Answer to Question 18

a. Year 4: $7680 \times 0.8 =$ **£6 144**. Year 5: $6144 \times 0.8 =$ **£4 915.20**. Each year the value is multiplied by 0.8.
b. First year: $15000 - 12000 =$ **£3 000**. Third year: $9600 - 7680 =$ **£1 920**. Subtract each year's value from the value at the start of that year.
c. After **5 years** (£4 915.20 is below £5 000, while £6 144 is above). Compare the values in the table with £5 000.
d. No: it is an exponential decay curve, falling steeply at first and then more slowly. The value falls by a smaller amount each year, so the graph is a curve, not a straight line.

## Answer to Question 19

a. $24 \div 5 =$ **4.8 hours** Divide 24 by the number of hoses.
b. $24 \div 2 =$ **12 hoses** Rearrange the formula: $n = 24 \div t$.
c. 3 hoses take 8 hours, which is more than 6, so **no**: at least 5 hoses would be needed. With 3 hoses the time is $24 \div 3 = 8$ hours, and 5 hoses would take 4.8 hours.
d. Time $\times$ number of hoses is constant (24), so twice as many hoses means half the time. This is inverse proportion: the product of the number of hoses and the time is always 24.

## Answer to Question 20

a. Week 8: $2^8 =$ **256**. Week 10: **1 024**. Each week Jo's savings double, so $2^8 = 256$ and $2^{10} = 1024$.
b. Week 7: Jo $128$, Sara $170$. Week 8: Jo $256$, Sara $180$. Jo first has more in **week 8**. Compare the two amounts in each week until Jo's is bigger.
c. Exponential growth starts slowly but eventually overtakes any linear growth. A linear graph adds the same amount each week, but an exponential graph

multiplies, so it grows ever faster.

d. Week 20: $2^{20} =$ **£1 048 576**. Week 30: $2^{30} =$ **£1 073 741 824**. Use the power key on a calculator for $2^{20}$ and $2^{30}$.

e. Nobody could find a billion pounds to save, so the doubling would have to stop. Real savings depend on real income, so the exponential model breaks down very quickly.