Test answers - Pythagoras' theorem

Year: Year 9 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. The unknown is the hypotenuse, so square and add: $7^2 + 24^2 = 49 + 576 = 625$, then $\sqrt{625} =$**25 cm**. (1 mark)
b. Square and add: $6^2 + 9^2 = 36 + 81 = 117$. Then $\sqrt{117} = 10.81\ldots$, which is **10.8 cm**. (1 mark)
c. The hypotenuse is known, so square and subtract: $15^2 - 9^2 = 225 - 81 = 144$, and $\sqrt{144} =$**12 cm**. (1 mark)
d. Square and subtract: $8^2 - 5^2 = 64 - 25 = 39$, and $\sqrt{39} = 6.24\ldots$, which is **6.2 cm**. (1 mark)

Parts (a) and (c) are Pythagorean triples (7, 24, 25 and 9, 12, 15), so their answers are exact. A quick sense check: the hypotenuse is always the longest side.

Answer to Question 2 (4 marks)

a. Compare the sum of the squares of the two shorter sides with the square of the longest side: $8^2 + 15^2 = 64 + 225 = 289$, and $17^2 = 289$ (1 mark). The two values are equal, so by the converse of Pythagoras' theorem the triangle **is** right-angled, with the right angle opposite the 17 cm side (1 mark).
b. $5^2 + 6^2 = 25 + 36 = 61$, but $8^2 = 64$ (1 mark). The values are different, so the triangle is **not** right-angled (1 mark).

In (b), 61 is less than 64, which means the longest side is too long for a right angle: the angle opposite it is obtuse. If the sum of the squares had been greater than 64, the largest angle would be acute instead.

Answer to Question 3 (4 marks)

a. Draw a right-angled triangle with AB as its hypotenuse. The horizontal difference is $8 - 2 = 6$ and the vertical difference is $9 - 1 = 8$ (1 mark). Then $AB = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} =$**10** units (1 mark).
b. Horizontal difference $5 - (-3) = 8$, vertical difference $4 - (-2) = 6$ (1 mark). Then $CD = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} =$**10** units (1 mark).

The sign of a difference does not matter because it is squared: $(-6)^2 = 36$ as well. Both segments are 6, 8, 10 triangles, a multiple of the 3, 4, 5 triple. Subtracting negative coordinates is where errors creep in, so write the brackets.

Answer to Question 4 (4 marks)

a. The ladder is the hypotenuse, the wall height is one short side and the distance from the wall is the other. So $d = \sqrt{6.5^2 - 6^2} = \sqrt{42.25 - 36}$ (1 mark) $= \sqrt{6.25} =$ **2.5 m** (1 mark).
b. The ladder is still 6.5 m long and still the hypotenuse. Subtract the square of the new distance, $3.9^2 = 15.21$: $h = \sqrt{42.25 - 15.21} = \sqrt{27.04}$ (1 mark) =**5.2 m** (1 mark).

Moving the foot out from 2.5 m to 3.9 m lowers the top by 0.8 m. Both triangles come from well-known triples: 2.5, 6, 6.5 is the 5, 12, 13 triple halved, and 3.9, 5.2, 6.5 is the 3, 4, 5 triple scaled by 1.3.

Answer to Question 5 (4 marks)

a. The diagonal is the hypotenuse of a right-angled triangle with sides 60 m and 80 m: $\sqrt{60^2 + 80^2} = \sqrt{3600 + 6400} = \sqrt{10000}$ =**100 m**. (1 mark)
b. Dan walks along two sides, $60 + 80 = 140$ m, while Eve walks 100 m. Dan walks $140 - 100$ =**40 m** further. (1 mark)
c. Time = distance ÷ speed. Eve: $100 \div 1.5 = 66.7$ s, which is **67 s** (1 mark). Dan: $140 \div 1.5 = 93.3$ s, which is **93 s** (1 mark).

Eve arrives about 27 seconds before Dan. The diagonal is always shorter than going round two sides, a fact known as the triangle inequality.