Answers - Pythagoras' theorem

Year: Year 9

Answer to Question 1

a. $c^2 = 36 + 64 = 100$, so $c =$ **10 cm** Square the two shorter sides and add them; the hypotenuse is the square root of the total. This is twice the 3, 4, 5 triangle.
b. $c^2 = 25 + 144 = 169$, so $c =$ **13 cm** 144 + 25 is 169, which is a square number, so the answer is exact.
c. $c^2 = 49 + 576 = 625$, so $c =$ **25 cm** The total 625 is 25^2, so the hypotenuse is a whole number.
d. $c^2 = 81 + 1600 = 1681$, so $c =$ **41 cm** Since $41^2 = 1681$, the hypotenuse is exactly 41 cm. The triple 9, 40, 41 is worth remembering.

Answer to Question 2

a. $c^2 = 16 + 49 = 65$, $c =$ **8.06 cm** 65 is not a square number, so use a calculator: $\sqrt{65} = 8.062\ldots$, which rounds to 8.06.
b. $c^2 = 6.25 + 36 = 42.25$, $c =$ **6.5 cm** $\sqrt{42.25} = 6.5$ exactly, so no rounding is needed.
c. $c^2 = 100 + 100 = 200$, $c =$ **14.14 cm** $\sqrt{200} = 14.142\ldots$; the triangle is isosceles, so the hypotenuse is $10\sqrt{2}$.
d. $c^2 = 10.24 + 20.25 = 30.49$, $c =$ **5.52 cm** $\sqrt{30.49} = 5.521\ldots$, which rounds to 5.52 cm.

Answer to Question 3

a. $a^2 = 169 - 25 = 144$, so $a =$ **12 cm** The hypotenuse is known, so subtract the square of the known side from the square of the hypotenuse.
b. $a^2 = 289 - 64 = 225$, so $a =$ **15 cm** Subtract, then take the square root: $\sqrt{225} = 15$. Check: $8^2 + 15^2 = 289$.
c. $a^2 = 100 - 49 = 51$, so $a =$ **7.14 cm** $\sqrt{51} = 7.141\ldots$, which rounds to 7.14 cm. The answer must be shorter than the hypotenuse.
d. $a^2 = 90.25 - 36 = 54.25$, so $a =$ **7.37 cm** $\sqrt{54.25} = 7.365\ldots$, which rounds to 7.37 cm.

Answer to Question 4

a. $25 + 144 = 169 = 13^2$. **Yes.** Always use the longest side as c. The squares of the two shorter sides add up to the square of the longest, so there is a right angle.
b. $49 + 64 = 113$ but $11^2 = 121$. **No.** The sum 113 is less than 121, so the angle opposite the longest side is bigger than a right angle.
c. $81 + 144 = 225 = 15^2$. **Yes.** 9, 12, 15 is three times the 3, 4, 5 triple, so it is right-angled.
d. $36 + 49 = 85$ but $9^2 = 81$. **No.** 85 is more than 81, so the largest angle is smaller than a right angle.

Answer to Question 5

a. $12, 16, 20$. Check: $144 + 256 = 400$. Multiply each number by 4. The check shows $12^2 + 16^2 = 20^2$.
b. $64 + 225 = 289$, so the missing side is $\sqrt{289} =$ **17**. The missing number is the hypotenuse, so add the squares and take the square root.
c. $625 - 576 = 49$, so the missing number is **7**. 25 is the hypotenuse, so subtract 24^2 from 25^2 and take the square root.
d. $400 + 441 = 841$ and $29^2 = 841$. It is a triple. The squares of the two smaller numbers add up to the square of the largest, so the three numbers form a Pythagorean triple.

Answer to Question 6

a. Differences 6 and 8: $\sqrt{36 + 64} =$ **10** The horizontal difference is $7 - 1 = 6$ and the vertical difference is $10 - 2 = 8$.
b. Differences 5 and 12: $\sqrt{25 + 144} =$ **13** The differences are $3 - (-2) = 5$ and $3 - (-9) = 12$; the signs do not matter once they are squared.
c. Differences 5 and 5: $\sqrt{50} =$ **7.07** $\sqrt{50} = 7.071\ldots$, which rounds to 7.07 units.
d. Differences 8 and 6: $\sqrt{64 + 36} =$ **10** The differences are $4 - (-4) = 8$ and $5 - (-1) = 6$, giving a distance of 10 units.

Answer to Question 7

a. $h^2 = 25 - 1.96 = 23.04$, so $h =$ **4.8 m** The ladder is the hypotenuse, so subtract the square of the distance from the wall.
b. $d^2 = 25 - 17.64 = 7.36$, so $d =$ **2.71 m** Now the height is known, so subtract 4.2^2 from 5^2. $\sqrt{7.36} = 2.712\ldots$.
c. At 4.8 m height the foot should be $4.8 \div 4 = 1.2$ m from the wall. It is 1.4 m, which is more than 1.2 m, so it is **safe** (the ladder is not too steep). Compare the distance from the wall with a quarter of the height: 1.4 m is a little more than 1.2 m.

d. $\sqrt{36 + 6.25} = \sqrt{42.25} =$ **6.5 m** Now the unknown is the hypotenuse, so add the squares of 6 and 2.5.

Answer to Question 8

a. $\sqrt{100^2 + 64^2} = \sqrt{14096} =$ **118.7 m** The diagonal is the hypotenuse of a right-angled triangle with sides 100 m and 64 m. $\sqrt{14096} = 118.72\ldots$
b. $118.7 \div 6 =$ **20 seconds** (19.8) Time = distance ÷ speed: $118.7 \div 6 = 19.8$, which rounds to 20 seconds.
c. $100 + 64 = 164$, so $164 - 118.7 =$ **45.3 m** further. Going along two sides is always longer than going straight across the diagonal.
d. Perimeter $= 2(100 + 64) = 328$ m, which needs **2 tins**. 328 m is more than one tin can mark, so she must round up to 2 tins.

Answer to Question 9

a. $\sqrt{88^2 + 50^2} = \sqrt{10244} =$ **101 cm** The diagonal is the hypotenuse: $\sqrt{10244} = 101.2\ldots$, which rounds to 101 cm.
b. $101.2 \div 2.54 = 39.8$, so **40 inches**. Divide by 2.54 because each inch is 2.54 cm. A 40-inch screen is a common size.
c. The scale factor is $11 \div 0.88 = 12.5$, so the height is $0.5 \times 12.5 =$ **6.25 m**. The width 88 cm is 0.88 m, and the cinema screen is an enlargement with scale factor 12.5.
d. $101.2 \times 12.5 = 1265$ cm = **12.65 m**. The diagonal is enlarged by the same scale factor of 12.5.

Answer to Question 10

a. $\sqrt{300^2 + 400^2} = \sqrt{250000} =$ **500 m** The shortcut is the hypotenuse of a triangle with sides 300 m and 400 m; this is the 3, 4, 5 triangle multiplied by 100.
b. Paths: $700 \div 1.25 =$ **560 s**. Shortcut: $500 \div 1.25 =$ **400 s**. Time = distance ÷ speed for each route; the paths are $300 + 400 = 700$ m long.
c. $560 - 400 =$ **160 seconds** (about 2 minutes 40 seconds). Subtract the shortcut time from the path time.
d. $45 > 30$, so **yes**, it is allowed. The width of 45 m is more than the minimum of 30 m.

Answer to Question 11

a. $\sqrt{1.8^2 + 1.2^2} = \sqrt{4.68} =$ **2.16 m** Each sloping side is the hypotenuse of a right-angled triangle with the pole and half the floor. $\sqrt{4.68} = 2.163\ldots$
b. $2 \times 2.16 \times 3 =$ **12.98 m²** (or 13.0 m²) Each sloping side of canvas is a rectangle 2.16 m by 3 m, and there are two of them.
c. $12.98 \times 6.50 = 84.4$, so **£84** Multiply the area by the price per square metre and round to the nearest pound.
d. $\sqrt{2.5^2 - 1.2^2} = \sqrt{4.81} =$ **2.19 m** The sloping side is the hypotenuse, so subtract 1.2^2 from 2.5^2. $\sqrt{4.81} = 2.193\ldots$

Answer to Question 12

a. $\sqrt{6^2 + 0.8^2} = \sqrt{36.64} =$ **6.05 m** The ramp surface is the hypotenuse of a triangle with sides 6 m and 0.8 m. $\sqrt{36.64} = 6.053\ldots$
b. $6.05 \times 18 =$ **£108.90** Multiply the length of the ramp by the cost per metre.
c. $6.05 \times 1.2 =$ **7.26 m²** The ramp surface is a rectangle 6.05 m long and 1.2 m wide.
d. Rise per 12 m would be $0.8 \times 2 = 1.6$ m. That is steeper than 1 m, so the ramp **does not meet** the 1 in 12 rule. A 1 in 12 rule allows a rise of only 0.5 m over 6 m, but this ramp rises 0.8 m.

Answer to Question 13

a. $\sqrt{64 + 225} = \sqrt{289} =$ **17 km** The journey east and south forms the two shorter sides of a right-angled triangle; 8, 15, 17 is a Pythagorean triple.
b. $17 \div 6 =$ **2.83 hours** Time = distance ÷ speed: $17 \div 6 = 2.833\ldots$ hours.
c. $0.83 \times 60 = 50$, so **2 hours 50 minutes**. Change the decimal part of an hour into minutes by multiplying by 60.
d. 17, so **yes**. The straight-line distance of 17 km is less than 20 km.

Answer to Question 14

a. $\sqrt{96^2 + 40^2} = \sqrt{10816} =$ **104 m** The flight path is the hypotenuse of a triangle with sides 96 m and 40 m.
b. 104, so **yes**. The flight of 104 m is less than the battery limit of 120 m.
c. $104 \div 10 =$ **10.4 seconds** Time = distance ÷ speed.
d. $\sqrt{72^2 + 30^2} = \sqrt{6084} = 78$ m, which is **shorter**. The second flight is the 12, 5, 13 triangle multiplied by 6: $\sqrt{6084} = 78$ m.

Answer to Question 15

a. A: $c = \sqrt{81 + 144} = 15$. B: $a = \sqrt{676 - 576} = 10$. C: $b = \sqrt{289 - 64} = 15$. D: $c = \sqrt{72} = 8.5$. Add the squares when finding the hypotenuse (A and D) and subtract them when finding a shorter side (B and C).
b. **Triangle D** (two equal sides, 6 and 6). An isosceles triangle has two equal sides; only D has $a = b$.
c. A: $9 + 12 + 15 =$ **36 cm**. D: $6 + 6 + 8.5 =$ **20.5 cm**. Add the three side lengths of each triangle.
d. $100 + 110.25 = 210.25 = 14.5^2$. **Yes.** The squares of the two shorter sides add up to the square of the longest side.

Answer to Question 16

a. The perpendicular height of an isosceles triangle bisects the base, because the two halves are congruent right-angled triangles. The line from the top to the middle of the base is a line of symmetry of the isosceles triangle.
b. $h = \sqrt{10^2 - 6^2} = \sqrt{64} =$ **8 cm** Use the right-angled triangle with hypotenuse 10 cm and base 6 cm.
c. $\frac{1}{2} \times 12 \times 8 =$ **48 cm²** Area of a triangle = $\frac{1}{2}$ × base × perpendicular height.
d. $10 + 10 + 12 =$ **32 cm** Add the three sides of the triangle.
e. Height $= \sqrt{100 - 25} = \sqrt{75} = 8.66$. Area $= \frac{1}{2} \times 10 \times 8.66 =$ **43.3 cm²** The height of an equilateral triangle splits the base into two halves of 5 cm; then use Pythagoras with hypotenuse 10 cm.

Answer to Question 17

a. $AB = \sqrt{6^2 + 8^2} =$ **10 km** The horizontal difference is $8 - 2 = 6$ and the vertical difference is $9 - 1 = 8$.
b. $BC = \sqrt{6^2 + 8^2} =$ **10 km** and $AC = 14 - 2 =$ **12 km**. B to C has differences 6 and 8 again. A and C have the same y-coordinate, so subtract the x-coordinates.
c. $AB = BC$, so it is **isosceles**. Two sides are equal, so the triangle is isosceles.
d. Perimeter $= 10 + 10 + 12 =$ **32 km**. Height from B is $9 - 1 = 8$, so area $= \frac{1}{2} \times 12 \times 8 =$ **48 km²**. AC is horizontal, so the height from B is the vertical distance from B to the line AC.
e. $32 \times 0.45 =$ **£14.40** Multiply the perimeter by the cost per kilometre.

Answer to Question 18

a. $\sqrt{40^2 + 30^2} = \sqrt{2500} =$ **50 cm** The base diagonal is the hypotenuse of a triangle

with sides 40 cm and 30 cm.

b. $\sqrt{50^2 + 120^2} = \sqrt{16900} =$ **130 cm** The base diagonal and the height make another right-angled triangle; its hypotenuse is the longest rod.

c. 1.3 m is exactly 130 cm, so a rod **just fits** (it touches opposite corners). Change 1.3 m into 130 cm before comparing.

d. No: 140 cm is greater than 130 cm. 140 cm is longer than the longest diagonal, so the rod cannot fit.

e. Surface area $= 2(40 \times 30 + 40 \times 120 + 30 \times 120) = 2(1200 + 4800 + 3600) = 19\,200$ cm^2. Cost $= 19200 \times 0.004 =$ **£76.80** A cuboid has three pairs of identical faces; find the area of each pair, add, then multiply by the cost.

Answer to Question 19

a. The horizontal difference is $16 - 10 = 6$ m. $\sqrt{6^2 + 8^2} =$ **10 m** Draw a right-angled triangle with the height of 8 m and the difference of the parallel sides, 6 m.

b. $10 + 16 + 8 + 10 =$ **44 m** Add all four sides: the two parallel sides, the perpendicular side and the sloping side.

c. $44 \times 9.50 =$ **£418** Multiply the perimeter by the cost per metre.

d. $\frac{1}{2}(10 + 16) \times 8 =$ **104 m²** Area of a trapezium = half the sum of the parallel sides × the distance between them.

e. $104 \times 4.80 =$ **£499.20** Multiply the area by the cost per square metre.

Answer to Question 20

a. $n = 2$: $3, 4, 5$. $n = 3$: $8, 6, 10$. $n = 4$: $15, 8, 17$. $n = 5$: $24, 10, 26$. Substitute each value of n into the three expressions.

b. $(n^2 - 1)^2 + (2n)^2 = n^4 - 2n^2 + 1 + 4n^2 = n^4 + 2n^2 + 1 = (n^2 + 1)^2$. Expand each bracket and collect like terms; both sides become $n^4 + 2n^2 + 1$.

c. $n = 6$: $35, 12, 37$. Check: $1225 + 144 = 1369 = 37^2$. Substitute $n = 6$: $36 - 1 = 35$, $2 \times 6 = 12$ and $36 + 1 = 37$.

d. $8, 6, 10$ is $2 \times (4, 3, 5)$, so it is twice the $3, 4, 5$ triangle. Dividing each number by 2 gives 4, 3, 5, the smallest Pythagorean triple.

e. Sides $15, 8, 17$: perimeter = **40**, area $= \frac{1}{2} \times 15 \times 8 =$ **60**. The two shorter sides 15 and 8 are perpendicular, so they are the base and height of the triangle.