Test answers - Probability: sample spaces and Venn diagrams

Year: Year 9 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. There are $4 + 5 + 1 = 10$ counters and 5 are blue: $P(\text{blue}) = \frac{5}{10} = \frac{1}{2}$. (1 mark)

b. One counter is green, so 9 of the 10 are not green: $P(\text{not green}) = 1 - \frac{1}{10} = \frac{9}{10}$. (1 mark)

c. A counter cannot be both red and green, so the events are mutually exclusive and the probabilities add: $\frac{4}{10} + \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$. (1 mark)

d. There are no yellow counters, so the event is impossible: $P(\text{yellow}) =$ **0**. (1 mark)

Parts (a) and (c) give the same answer because "red or green" is exactly "not blue", the complement of blue.

Answer to Question 2 (4 marks)

a. A sample space of two dice has $6 \times 6 = 36$ equally likely outcomes. The totals of 8 are (2, 6), (3, 5), (4, 4), (5, 3) and (6, 2), which is five outcomes (1 mark). The probability is $\frac{5}{36}$ (1 mark).

b. A difference of 2 comes from (1, 3), (2, 4), (3, 5), (4, 6) and their reverses, (3, 1), (4, 2), (5, 3), (6, 4), so there are eight ordered pairs (1 mark). The probability is $\frac{8}{36} = \frac{2}{9}$ (1 mark).

(4, 4) is counted only once in (a), because reversing it gives the same outcome. Listing (2, 6) but forgetting (6, 2) is the most common error; a 6 by 6 grid of totals or differences prevents it.

Answer to Question 3 (4 marks)

a. Fill in the overlap first: 5 do both. Swim only $= 17 - 5 = 12$; cycle only $= 12 - 5 = 7$. Together $12 + 5 + 7 = 24$, so neither $= 30 - 24 = 6$. (2 marks for all four regions, 1 mark for two)

b. Six pupils do neither activity out of 30: $\frac{6}{30} = \frac{1}{5}$. (1 mark)

c. "At least one" is the complement of "neither": $1 - \frac{1}{5} = \frac{24}{30} = \frac{4}{5}$. (1 mark)

Adding $17 + 12 = 29$ double-counts the 5 pupils who do both, which would wrongly leave only 1 pupil outside both circles. Check: $12 + 5 + 7 + 6 = 30$, so every pupil appears in exactly one region of the Venn diagram.

Answer to Question 4 (4 marks)

a. The three colours cover every outcome, so their probabilities add to 1: $x = 1 - 0.2 - 0.5 =$**0.3**. (1 mark)
b. "Not blue" is the complement of blue: $1 - 0.5 =$**0.5**. (1 mark)
c. Expected number = number of spins × probability: $150 \times 0.3 =$**45** greens. (1 mark)
d. Red and green cannot happen on the same spin, so add their probabilities: $0.2 + 0.3 =$**0.5**. (1 mark)

Parts (b) and (d) agree, because not blue means red or green. In 150 spins we would also expect about 30 reds and 75 blues; together with the 45 greens these make 150. The actual counts will vary a little from these expected values.

Answer to Question 5 (4 marks)

a. Only one of the six equally likely faces is a 6: $P(\text{win}) = \frac{1}{6}$. (1 mark)
b. Expected wins = number of plays × probability: $300 \times \frac{1}{6} =$**50**. (1 mark)
c. Each of the 50 expected wins costs the fair £5: $50 \times 5 =$**£250** in prizes. (1 mark)
d. The fair takes $300 \times £1 = £300$ and expects to pay out £250, so its expected profit is $300 - 250 =$**£50**. (1 mark)

On average each play gives the fair about 17p, since $50 \div 300 \approx 0.17$. For the game to be fair, the prize would have to be £6, so that the expected payout equals the £300 taken.