Answers - Probability: sample spaces and Venn diagrams

Year: Year 9

Answer to Question 1

a. One outcome out of 8: $\frac{1}{8}$ The spinner is fair, so each of the 8 sections is equally likely and only one section shows 5.

b. Even numbers are 2, 4, 6, 8: $\frac{4}{8} = \frac{1}{2}$ Four of the eight numbers are even, so the probability simplifies to one half.

c. Primes are 2, 3, 5, 7: $\frac{4}{8} = \frac{1}{2}$ A prime number has exactly two factors; 1 is not prime. Four of the eight numbers are prime.

d. 7 and 8: $\frac{2}{8} = \frac{1}{4}$ Only 7 and 8 are greater than 6, so two outcomes out of eight, which simplifies to one quarter.

Answer to Question 2

a. $1 - 0.35 = \mathbf{0.65}$ It either rains or it does not, so the two probabilities add up to 1.

b. $1 - \frac{3}{8} = \frac{5}{8}$ Subtract from 1, written as $\frac{8}{8}$, to get the probability that the counter is not red.

c. $1 - 0.2 = \mathbf{0.8}$ Assuming a draw is not possible, winning and losing are the only outcomes, so they add up to 1.

d. $1 - \frac{7}{40} = \frac{33}{40}$ Write 1 as $\frac{40}{40}$ and subtract $\frac{7}{40}$.

Answer to Question 3

a. $0.3 + 0.45 + 0.15 + x = 1$, so $0.9 + x = 1$ and $x = \mathbf{0.1}$ The four colours are the only possible outcomes, so their probabilities add up to 1.

b. $0.3 + 0.45 = \mathbf{0.75}$ Red and blue cannot happen at the same time, so add their probabilities.

c. $1 - 0.15 = \mathbf{0.85}$ Not green is the complement of green, so subtract from 1. Check: $0.3 + 0.45 + 0.1 = 0.85$.

d. $0.45 + 0.1 = \mathbf{0.55}$ Blue and yellow are mutually exclusive, so add 0.45 and the value of x from part a).

Answer to Question 4

a. Six 7s in the table: $\frac{6}{36} = \frac{1}{6}$ The 7s lie on a diagonal of the table, one in each row, so there are 6 of them out of 36.

b. Doubles are $(1,1)$ to $(6,6)$: $\frac{6}{36} = \frac{1}{6}$ Each score can be doubled in exactly one way, giving 6 doubles out of 36 outcomes.

c. Totals 11 and 12: $2 + 1 = 3$, so $\frac{3}{36} = \frac{1}{12}$ A total of 11 can happen in 2 ways and 12 in only 1 way.

d. Totals 2 and 3: $1 + 2 = 3$, so $\frac{3}{36} = \frac{1}{12}$ A total of 2 can happen in 1 way and 3 in 2 ways, giving 3 outcomes out of 36.

Answer to Question 5

a. H1, H2, H3, T1, T2, T3 Pair each side of the coin with each number on the spinner, so that no outcome is missed.

b. There are **6** outcomes (2×3). There are 2 coin outcomes and 3 spinner outcomes, so $2 \times 3 = 6$ combinations.

c. One outcome (H2): $\frac{1}{6}$ All 6 outcomes are equally likely, and only H2 matches.

d. T1 and T3: $\frac{2}{6} = \frac{1}{3}$ The odd numbers are 1 and 3, so two of the six outcomes match.

Answer to Question 6

a. $A \cap B = \{6, 12\}$. $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$. The intersection contains numbers in both sets: even multiples of 3. The union contains numbers in at least one set.

b. $P(A) = \frac{6}{12} = \frac{1}{2}$. $P(A \cap B) = \frac{2}{12} = \frac{1}{6}$. There are 6 even numbers out of 12, and 2 numbers in the intersection.

c. Neither: 1, 5, 7, 11. $\frac{4}{12} = \frac{1}{3}$ Numbers outside both circles are odd and not multiples of 3: there are four of them.

d. $n(A) = 6$, $n(B) = 4$, $n(A \cap B) = 2$: $6 + 4 - 2 = 8 = n(A \cup B)$. Correct. The formula subtracts the overlap because it was counted twice, once in A and once in B.

Answer to Question 7

a. $\frac{5}{200} = \frac{1}{40}$ Each ticket has the same chance, and Maya has 5 of the 200 tickets.
b. $\frac{12}{200} = \frac{3}{50}$ Ali has 12 of the 200 tickets; the fraction simplifies by dividing by 4.
c. $\frac{5}{200} + \frac{12}{200} = \frac{17}{200}$. Only one ticket wins, so the events are mutually exclusive. Only one ticket is drawn, so Maya and Ali cannot both win, and we may add their probabilities.
d. $1 - \frac{17}{200} = \frac{183}{200}$ Someone else winning is the complement of Maya or Ali winning.
e. $200 - 50 =$ **£150** The school receives £1 for each of the 200 tickets and pays out one prize of £50.

Answer to Question 8

a. $1 - 0.6 =$ **0.4** A day is either sunny or not sunny, so the probabilities add up to 1.
b. $20 \times 0.6 =$ **12** Expected number = number of trials × probability.
c. Yes, as long as rain is part of the 0.4: the not-sunny days are 0.4, which is more than 0.25. Rainy days must fit inside the not-sunny days, so the probability of rain cannot be more than 0.4.
d. $50 \times 0.4 =$ **20** Multiply the number of Saturdays by the probability of not sunny.

Answer to Question 9

a. $P(\text{red}) = \frac{8}{20} = \frac{2}{5}$; $P(\text{orange}) = \frac{2}{20} = \frac{1}{10}$ There are 20 sweets altogether; 8 are red and 2 are orange.
b. $\frac{8+6}{20} = \frac{14}{20} = \frac{7}{10}$ A sweet cannot be red and green at once, so add the 8 red and 6 green sweets.
c. $1 - \frac{1}{10} = \frac{9}{10}$ Use the complement: the probability of orange is $\frac{1}{10}$.
d. Now there are 6 red out of 18 sweets: $\frac{6}{18} = \frac{1}{3}$ After eating two red sweets there are 2 fewer red sweets and 2 fewer sweets in total.

Answer to Question 10

a. $\frac{6}{36} = \frac{1}{6}$ There are 6 doubles among the 36 equally likely outcomes.

b. Totals 9, 10, 11, 12 occur $4+3+2+1=10$ times: $\frac{10}{36}=\frac{5}{18}$ Count the outcomes in the table with a total of 9, 10, 11 or 12.
c. Pairs $(2,6),(6,2),(3,4),(4,3)$: $\frac{4}{36}=\frac{1}{9}$ Only 2×6 and 3×4 give 12, and each can happen in either order.
d. $72\times\frac{1}{6}=$ **12 extra turns** Expected number = number of rolls × probability of a double.

Answer to Question 11

a. Pizza-juice, pizza-water, pasta-juice, pasta-water, salad-juice, salad-water. Work systematically: take each main course and pair it with each drink.
b. **6** outcomes. There are 3 main courses and 2 drinks, so $3\times2=6$ possible lunches.
c. One outcome: $\frac{1}{6}$ Each lunch is equally likely, and only one of the six is pizza with water.
d. Two outcomes (salad-juice, salad-water): $\frac{2}{6}=\frac{1}{3}$ Salad appears with each of the two drinks.
e. Pasta-juice, pasta-water, pizza-juice, salad-juice: $\frac{4}{6}=\frac{2}{3}$ Count every lunch that contains pasta or juice or both; pasta-juice is counted only once.

Answer to Question 12

a. Football only $=18-5=13$. Hockey only $=12-5=7$. The 5 pupils who play both are inside both circles, so subtract them from each total.
b. $13+5+7=25$, so neither $=30-25=$ **5**. Add the three regions inside the circles and subtract from the class total.
c. $\frac{5}{30}=\frac{1}{6}$ Five of the 30 pupils play both sports.
d. $\frac{25}{30}=\frac{5}{6}$ Football or hockey means at least one sport: $13+5+7=25$ pupils.
e. $\frac{13+7}{30}=\frac{20}{30}=\frac{2}{3}$ Exactly one sport means football only or hockey only, but not both.

Answer to Question 13

a. $1-0.15=$ **0.85** Winning and not winning are complementary, so subtract 0.15 from 1.

b. $200 \times 0.15 = $ **30 prizes** Expected number of prizes = 200 × 0.15.
c. $30 \times 4 = $ **£120** Each of the 30 expected prizes is worth £4.
d. Takings £200 minus £120 = **£80** The fair takes £1 for each spin, so it takes £200 and pays out about £120.
e. £80 is below £100, so the organiser **would need about 250 spins** ($250 - 37.5 \times 4 = 100$). Each spin gives an expected profit of $1 - 0.15 \times 4 = 0.40$ pounds, so £100 needs 250 spins.

Answer to Question 14

a. $\frac{13}{52} = \frac{1}{4}$ There are 13 hearts among the 52 cards.
b. $\frac{4}{52} = \frac{1}{13}$ There is one king in each of the four suits.
c. $\frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$ A card cannot be a king and a queen at once, so add the 4 kings and the 4 queens.
d. $1 - \frac{1}{4} = \frac{3}{4}$ There are 13 spades, so 39 cards are not spades, which is $\frac{39}{52} = \frac{3}{4}$.
e. $104 \times \frac{1}{4} = $ **26** Expected number = number of trials × probability of a heart.

Answer to Question 15

a. Boys $= 50$, girls $= 50$; walk $= 38$, bus $= 42$, car $= 20$. Add across each row to find the totals for boys and girls, and down each column for each type of travel.
b. $\frac{38}{100} = $ **0.38** 38 of the 100 pupils walk.
c. $\frac{24}{100} = $ **0.24** Look at the cell where the girls' row meets the bus column: 24 out of 100.
d. $\frac{16}{50} = $ **0.32** This time the total is only the 50 girls, of whom 16 walk.
e. $\frac{20}{100} \times 1200 = $ **240 pupils** The sample suggests that 20% of pupils travel by car; 20% of 1 200 is 240.

Answer to Question 16

a. $3 \times 4 = $ **12** outcomes. Spinner A has 3 outcomes and spinner B has 4, so $3 \times 4 = 12$.
b. Totals of 5: 3 outcomes, so $\frac{3}{12} = \frac{1}{4}$ A total of 5 comes from (1,4), (2,3) and (3,2).

c. Even totals: 2, 4, 4, 6, 4, 6, so 6 outcomes: $\frac{1}{2}$ Count the even totals in the grid: 6 out of 12.

d. Totals of 6, 6 and 7: $\frac{3}{12} = \frac{1}{4}$ The totals greater than 5 are 6, 6 and 7.

e. $120 \times \frac{1}{4} = \mathbf{30}$ Expected number = number of trials × probability.

Answer to Question 17

a. French only $= 16$, both $= 6$, Spanish only $= 9$, neither $= 40 - 31 = 9$. Fill in the overlap first: 6 study both, so French only is $22 - 6$ and Spanish only is $15 - 6$.

b. $\frac{9}{40}$ Nine of the 40 students are outside both circles.

c. $\frac{16 + 9}{40} = \frac{25}{40} = \frac{5}{8}$ Exactly one language means French only or Spanish only.

d. Among the 22 French students, 6 also study Spanish: $\frac{6}{22} = \frac{3}{11}$ The question restricts the choice to the 22 French students, so the denominator is 22.

e. $\frac{9}{40} \times 200 = \mathbf{45\ students}$ Expected number = 200 × the probability of neither language.

Answer to Question 18

a. $P(7) = \frac{6}{36} = \frac{1}{6}$ and $P(\text{double}) = \frac{6}{36} = \frac{1}{6}$. A total of 7 is odd, and the total of a double is even, so they cannot happen together. Look at the 36-outcome grid for two dice. Since a double always gives an even total, a total of 7 and a double are mutually exclusive.

b. $\frac{1}{6} \times 4 + \frac{1}{6} \times 2 = \frac{6}{6} = \mathbf{£1}$ Multiply each prize by its probability and add.

c. Yes: the expected winnings equal the £1 cost, so the game is **fair** in the long run. A game is fair when the expected winnings equal the cost of playing.

d. Takings $= 360 \times 1 = \mathbf{£360}$. Expected payout $= 60 \times 4 + 60 \times 2 = 240 + 120 = \mathbf{£360}$. Expected wins of each type: $360 \times \frac{1}{6} = 60$. The payout equals the takings, so the fair expects no profit.

Answer to Question 19

a. $2 \times 2 \times 2 = \mathbf{8}$ outcomes. Each coin has 2 outcomes, so three coins have $2 \times 2 \times 2$ outcomes.

b. Three outcomes out of 8: $\frac{3}{8}$ HHT, HTH and THH each have exactly two heads.

c. Everything except HHH: $1 - \frac{1}{8} = \frac{7}{8}$ At least one tail is the complement of no tails, which is HHH.

d. HHH and TTT: $\frac{2}{8} = \frac{1}{4}$ All the same means all heads or all tails.

e. $80 \times \frac{3}{8} = \mathbf{30}$ Expected number = 80 × the probability of exactly 2 heads.

Answer to Question 20

a. Pizza only $= 18 - x$; pasta only $= 20 - x$. The x pupils who like both are inside both circles, so subtract x from each total.

b. $(18 - x) + x + (20 - x) + 2 = 30$, so $40 - x = 30$. The 2 pupils who like neither are outside both circles, and all four regions add up to 30.

c. $x = \mathbf{10}$ Collect the terms: $40 - x = 30$. Check: $8 + 10 + 10 + 2 = 30$.

d. Pizza only $= 8$: $\frac{8}{30} = \frac{4}{15}$ Pizza only is $18 - 10 = 8$ pupils out of 30.

e. Of the 20 who like pasta, 10 like pizza: $\frac{10}{20} = \frac{1}{2}$ The choice is limited to the 20 pasta lovers, and 10 of them are in the overlap.