Lesson - Percentage change and simple interest

Year: Year 9

This lesson covers multipliers, percentage increase and decrease, finding a percentage change, reverse percentages, repeated change and simple interest.

Starter activity

A jacket costs £50. Shop A takes 20% off. Shop B takes £10 off. Work out both prices. Now the jacket costs £200: compare again. Which shop is better now? Then try this: a price of £100 goes up by 10% and then down by 10%. Predict the final price, then work it out. Were you right?

Percentages as multipliers

Definition: multiplier

A percentage is a number out of 100. A **multiplier** is the number you multiply by to change an amount by a percentage. To increase by 15% you end up with $100\% + 15\% = 115\%$, so you multiply by 1.15. To decrease by 15% you keep $100\% - 15\% = 85\%$, so you multiply by 0.85.

Change	Calculation	Multiplier
Increase by 8%	100% + 8% = 108%	1.08
Decrease by 8%	100% − 8% = 92%	0.92
Increase by 2.5%	100% + 2.5% = 102.5%	1.025
Decrease by 35%	100% − 35% = 65%	0.65
Increase by 100%	200%	2

Percentage increase and decrease

Worked example 1: increase

Increase £80 by 15%.

Multiplier 1.15. $80 \times 1.15 = £92$. By finding the percentage: 10% is £8, 5% is £4, so 15% is £12 and $80 + 12 = £92$. The multiplier method is quicker and is essential for harder problems.

Worked example 2: decrease

A coat costs £60. In a sale it is reduced by 35%. Find the sale price.

Multiplier 0.65. $60 \times 0.65 = £39$.

Finding a percentage change

Rule: percentage change

$$\text{percentage change} = \frac{\text{actual change}}{\text{original}} \times 100\%$$

Always divide by the **original** value, which is the value before the change happened.

Worked example 3: increase

A console costs £250 and later costs £290. Find the percentage increase.

The change is $290 - 250 = 40$. $\frac{40}{250} \times 100 = 16\%$ increase.

Worked example 4: decrease

A phone costs £240 and is reduced to £204. Find the percentage decrease.

The change is £36. $\frac{36}{240} \times 100 = 15\%$ decrease.

Original value problems (reverse percentages)

Method: reverse percentages

1. Write the multiplier for the change.
2. **Divide** the after-value by the multiplier to get the original.
3. Check by multiplying forwards.

Worked example 5: after an increase

After a 20% increase a price is £36. Find the original price.

The multiplier is 1.2. Original $= 36 \div 1.2 = £30$.

Worked example 6: after a sale

In a sale all prices are cut by 15%. A jacket costs £51 in the sale. Find the original price.

The multiplier is 0.85. Original $= 51 \div 0.85 = £60$.

Watch out: finding 15% of the new price

A very common mistake is to find 15% of £51 and add it on. That is wrong, because the 15% was taken off the *original* price, not off £51.

Repeated percentage change

Rule: repeated change

If a quantity changes twice, multiply the multipliers. A £400 television goes up by 10% and then comes down by 10%: $400 \times 1.1 \times 0.9 = 400 \times 0.99 = £396$. It is not back to £400, because the second 10% is taken from a bigger number.

Simple interest

Definition: simple interest

Interest is the extra money paid when you save or borrow. With **simple interest** the same amount is paid every year, calculated on the original amount only (the **principal**).

Rule: simple interest formula

$$I = \frac{P \times R \times T}{100}$$

P is the principal in pounds, R is the yearly percentage rate and T is the time in years.

Worked example 7: interest earned

£500 is saved at 3% simple interest for 4 years. Find the interest and the total.

Interest each year is 3% of £500 = £15, so over 4 years $4 \times 15 = £60$. Using the formula: $\frac{500 \times 3 \times 4}{100} = 60$. The total is $500 + 60 = £560$.

Worked example 8: finding the rate

Interest of £84 is earned on £700 over 3 years. Find the rate.

Rearrange: $R = \frac{100 \times I}{P \times T} = \frac{100 \times 84}{700 \times 3} = \frac{8400}{2100} = 4\%$.

Note: choosing a method

Do I know the original and want the new value? Multiply by the multiplier. Do I know the new value and want the original? Divide by the multiplier. Do I know both values and want the percentage change? Use change divided by original. Writing the question type down first prevents most errors.

Watch out: common mistakes

- Dividing by the new value instead of the original value when finding percentage change.
- Using 0.15 instead of 0.85 as the multiplier for a 15% decrease.
- Adding the percentages for two changes (10% up then 10% down is not 0%).
- In simple interest, forgetting to multiply by the number of years, or forgetting to add the interest to the principal when the total is asked for.

Worked example 9: a decimal percentage

Increase £480 by 7.5%.

The multiplier is $1 + 0.075 = 1.075$. So $480 \times 1.075 = 516$. The new amount is £516.

Worked example 10: compound growth with a power

£1 000 earns 5% compound interest each year. Find the value after 10 years.

Each year the value is multiplied by 1.05. After 10 years:
$1000 \times 1.05^{10} = 1000 \times 1.628894\ldots =$ £1 628.89 to the nearest penny. Simple interest would give only $1000 + 10 \times 50 =$ £1 500, so compound interest earns £128.89 more.

Worked example 11: percentage profit

A shop buys a bike for £160 and sells it for £212. Find the percentage profit.

Profit $= 212 - 160 = 52$. Percentage profit $= 52 \div 160 \times 100 = 32.5\%$. Always divide by the *original* amount, which here is the cost price.

Worked example 12: finding the time

How long does it take £400 to earn £60 simple interest at 3% per year?

$60 = \dfrac{400 \times 3 \times T}{100} = 12T$, so $T = 60 \div 12 = 5$. It takes 5 years.

Worked example 13: two-step reverse percentage

A pair of boots is reduced by 30% in a sale, and then costs £52.50. Find the original price and the amount saved.

The sale price is 70% of the original, so the multiplier is 0.7. Original $= 52.50 \div 0.7 = 75$. The boots cost £75 before the sale. The saving is $75 - 52.50 = 22.50$, which is £22.50. Check: 30% of 75 is 22.50, which agrees.

Method: any percentage problem

1. Decide whether the quantity is going up, going down or being compared.
2. Write the multiplier, or the fraction change over original.
3. Multiply to find the new value, divide to find the original value.
4. Check that the answer is sensible: an increase must give a bigger number.

Note: which method?

Use **multipliers** for increases and decreases, for repeated change and for reverse percentages (divide). Use **change over original** to find a percentage change. Use $I = PRT/100$ only for simple interest. If the problem says "per year" and the interest is added each year, think compound; if it says "simple", the interest is always worked out on the original sum.

Watch out: percentages of different amounts

- An increase of 10% followed by a decrease of 10% does not return to the start, because the second percentage is of a different amount.
- To undo a percentage change, divide by the multiplier; do not subtract or add the same percentage.
- Percentage change is always compared with the *original* amount, not the new one.

Did you know?

Interest is thousands of years old: clay tablets from ancient Babylon, about 4 000 years ago, record loans of silver and grain with interest. Today the Bank of England sets a base rate that affects the interest on savings and loans across the whole of the UK.

Percentages in everyday money

Worked example 14: comparing two savings accounts

Account A pays 4% simple interest per year. Account B pays 3.8% compound interest per year. Which gives more on £2 000 after 5 years?

For account A the interest is the same every year: $2000 \times 4 \times 5 \div 100 = 400$, so the total is £2 400. For account B we multiply by 1.038 once for every year: $2000 \times 1.038^5 = 2409.97$ to the nearest penny. Even though the rate is lower,

compound interest earns interest on the interest, so account B gives about £10 more after 5 years. Over a shorter time, simple interest at the higher rate would win, so always check the number of years in the question.

Worked example 15: finding the percentage of a percentage

In a school, 60% of pupils have a bus pass, and 25% of those with a pass also have a bike. What percentage of all pupils have both?

Find 25% of 60%. Using decimals, $0.25 \times 0.60 = 0.15$, so 15% of all pupils have both a pass and a bike. If the school has 840 pupils, that is $840 \times 0.15 = 126$ pupils. Multiplying the decimals works for the same reason as using repeated multipliers.

Note: percentage points

When a rate changes, newspapers sometimes talk about percentage points. If an interest rate rises from 4% to 5%, it has gone up by 1 percentage point, but the percentage increase in the rate is $1 \div 4 \times 100 = 25\%$. Read questions carefully to see which one is wanted.

Otto's tip

Write the multiplier before you press any buttons. If you want to go back to the original, divide by the same multiplier. Hoot hoot!