Answers - Percentage change and simple interest

Year: Year 9

Answer to Question 1

a. $1 + 0.20 =$ **1.2** An increase means we keep the whole amount (1) and add 20% (0.20).
b. $1 - 0.15 =$ **0.85** A decrease means we keep 100% minus 15%, which is 85%, written as 0.85.
c. $1 + 0.075 =$ **1.075** Write 7.5% as a decimal by dividing by 100: 0.075. Then add it to 1.
d. $1 - 0.04 =$ **0.96** Take 4% (0.04) away from 1, because 96% of the amount is left.

Answer to Question 2

a. $80 \times 1.25 =$ **£100** The multiplier for a 25% increase is 1.25. The new amount is £100.
b. $64 \times 0.65 =$ **£41.60** A 35% decrease leaves 65%, so the multiplier is 0.65. The new amount is £41.60.
c. $240 \times 1.12 =$ **£268.80** The multiplier for a 12% increase is 1.12. The new amount is £268.80.
d. $1500 \times 0.92 =$ **£1 380** An 8% decrease leaves 92%, so the multiplier is 0.92. The new amount is £1 380.

Answer to Question 3

a. Change $= 12$. $12 \div 40 \times 100 =$ **30% increase** The change is $52 - 40 = 12$; divide by the original value 40, not by the new value.
b. Change $= 20$. $20 \div 80 \times 100 =$ **25% decrease** The value has gone down by 20, which is a quarter of the original 80.
c. Change $= 25$. $25 \div 250 \times 100 =$ **10% increase** The change of 25 is one tenth of the original 250.
d. Change $= 4.5$. $4.5 \div 18 \times 100 =$ **25% decrease** The change is $18 - 13.5 = 4.5$; divide by the original 18 and multiply by 100.

Answer to Question 4

a. $60 \div 1.2 =$ **£50** After a 20% increase the price is 120% of the original, so divide by 1.2. Check: $50 \times 1.2 = 60$.

b. $68 \div 0.85 =$ **£80** After a 15% decrease the price is 85% of the original, so divide by 0.85. Check: $80 \times 0.85 = 68$.
c. $150 \div 1.25 =$ **120 kg** The new mass is 125% of the original, so divide by 1.25. Check: $120 \times 1.25 = 150$.
d. $45 \div 0.9 =$ **50 pupils** The new number is 90% of the original, so divide by 0.9. Check: $50 \times 0.9 = 45$.

Answer to Question 5

a. $200 \times 1.1 \times 1.1 =$ **£242** Apply the multiplier 1.1 once for each increase: £200 becomes £220, then £242.
b. The second 10% is of £220, not of £200, so the total increase is 21%, not 20%. The single multiplier is $1.1 \times 1.1 = 1.21$, which is a 21% increase.
c. $1000 \times 0.8 \times 0.8 =$ **£640** The multiplier 0.8 is used twice: £1 000 becomes £800, then £640. This is a 36% decrease, not 40%.
d. $400 \times 1.05^3 = 400 \times 1.157625 =$ **£463.05** Use the power key: $1.05^3 = 1.157625$. Multiply by 400 and round to the nearest penny.

Answer to Question 6

a. $I = 500 \times 4 \times 3 \div 100 =$ **£60**. Total $= 500 + 60 =$ **£560**. Substitute $P = 500$, $R = 4$ and $T = 3$; the interest is £20 each year for 3 years.
b. $1200 \times 2.5 \times 4 \div 100 =$ **£120** Substitute $P = 1200$, $R = 2.5$ and $T = 4$; the interest is £30 each year.
c. $96 = 800 \times R \times 3 \div 100$, so $R = 96 \times 100 \div 2400 =$ **4%** Substitute the known values and rearrange the formula to make R the subject. Check: $800 \times 4 \times 3 \div 100 = 96$.
d. $90 = 600 \times 5 \times T \div 100$, so $T = 90 \times 100 \div 3000 =$ **3 years** Rearrange the formula to make T the subject. Check: £30 a year for 3 years is £90.

Answer to Question 7

a. $85 \times 0.70 =$ **£59.50** A 30% reduction leaves 70% of the price, so the multiplier is 0.70. The trainers cost £59.50.
b. $48 \times 0.85 =$ **£40.80**. Saved $= 48 - 40.80 =$ **£7.20**. A 15% reduction leaves 85%, so the multiplier is 0.85. The boots cost £40.80, a saving of £7.20.
c. $60 \times 0.80 \times 0.90 =$ **£43.20** Multiply by 0.80 for the first reduction and then by 0.90 for the second. The coat costs £43.20.
d. $60 \times 0.70 = £42$, which is different from £43.20, so **no**. The second 10% is taken from the already reduced price. The single multiplier is $0.8 \times 0.9 = 0.72$, which is a 28% reduction, not 30%.

Answer to Question 8

a. Multiplier 1.035; new pay $= 1200 \times 1.035 =$ **£1 242** A 3.5% rise means the multiplier is $1 + 0.035 = 1.035$. His new pay is £1 242 a month.
b. Rise per month $= £42$, so per year $42 \times 12 =$ **£504** The rise is $1242 - 1200 = 42$ pounds per month, and there are 12 months in a year.
c. $1242 \times 0.04 =$ **£49.68** Multiply the new monthly pay by 0.04 to find 4%.
d. $1242 - 49.68 =$ **£1 192.32** Subtract the pension payment from the new monthly pay.

Answer to Question 9

a. $75 \times 1.2 =$ **£90** Adding 20% VAT means the multiplier is 1.2. The phone costs £90 including VAT.
b. $132 \div 1.2 =$ **£110** The £132 is 120% of the price before VAT, so divide by 1.2. Check: $110 \times 1.2 = 132$.
c. $132 - 110 =$ **£22** The VAT is the difference between the two prices: £22.
d. 20% of £132 is £26.40, which would give £105.60. But the 20% was added to the smaller original price, so you must divide by 1.2. The 20% was a percentage of the original price, not of £132, so subtracting 20% of £132 removes too much.

Answer to Question 10

a. $12000 \times 1.02 =$ **12 240** A 2% increase means the multiplier is 1.02. After 1 year the population is 12 240.
b. $12000 \times 1.02^2 = 12\,484.8 =$ **12 485** Apply the multiplier twice, or use 1.02^2, then round to a whole number of people.
c. $12\,484.8 - 12\,000 = 484.8$, so about **485 people**. Subtract the starting population from the population after 2 years: about 485 more people.
d. $1.02^2 = 1.0404$, so **4.04%** The single multiplier is 1.0404, which is an overall increase of 4.04%, a little more than $2 + 2 = 4\%$.

Answer to Question 11

a. $240000 \times 1.05 =$ **£252 000** A 5% rise means the multiplier is 1.05. After the first year the house is worth £252 000.
b. $252000 \times 0.95 =$ **£239 400** A 5% fall means the multiplier is 0.95, applied to the new value.
c. **Less**, by $240\,000 - 239\,400 =$ **£600**. The single multiplier is $1.05 \times 0.95 = 0.9975$, which is a 0.25% fall overall.
d. The 5% decrease is taken from the bigger value (£252 000), so it removes more than the 5% that was added to £240 000. The fall is 5% of a larger amount, so it is bigger than the rise. Therefore the value ends slightly lower.

Answer to Question 12

a. $800 \times 3 \times 5 \div 100 =$ **£120**; total **£920**. Substitute $P = 800$, $R = 3$ and $T = 5$: Tara earns £24 a year, so £120 in 5 years.
b. $800 \times 2.5 \times 6 \div 100 =$ **£120** Substitute $P = 800$, $R = 2.5$ and $T = 6$: Omar earns £20 a year, so £120 in 6 years.
c. Tara after 6 years: $800 \times 3 \times 6 \div 100 = 144$, so Tara has £944 and Omar £920, so **Tara** has **£24** more. Work out both totals after the same number of years before comparing them.
d. Omar earns £20 per year, so $150 \div 20 = 7.5$, so after **8 years**. After 7 years Omar has only £140 of interest, so the first whole year with at least £150 is year 8.

Answer to Question 13

a. Change $= 3700$. $3700 \div 18500 \times 100 =$ **20%** The change is $22\,200 - 18\,500 = 3700$; divide by last season's figure, the original.
b. $22200 \times 0.85 =$ **18 870** A 15% fall leaves 85%, so multiply by 0.85. The expected attendance is 18 870.
c. $22200 \times 24 =$ **£532 800** Multiply the number of fans by the price of a ticket: £532 800.
d. $18870 \times 24 =$ **£452 880** Use the attendance from part b). The income falls by £79 920, which is also 15%.

Answer to Question 14

a. Change $= 0.36$. $0.36 \div 2.40 \times 100 =$ **15%** The increase is $2.76 - 2.40 = 0.36$; divide by the original fare of £2.40.
b. $2.76 \times 1.1 = 3.036 =$ **£3.04** A 10% rise means the multiplier is 1.1; £3.036 rounds to £3.04.
c. $3.30 \div 1.1 =$ **£3.00** £3.30 is 110% of the old price, so divide by 1.1. Check: $3.00 \times 1.1 = 3.30$.
d. $3.036 \div 2.40 = 1.265$, so **26.5%** increase overall. Divide the final fare by the original fare to find the single multiplier, then subtract 1.

Answer to Question 15

a. A: $90 \times 0.75 =$ **£67.50**. B: $90 \times \frac{2}{3} =$ **£60**. C: $90 - 28 =$ **£62**. Use a multiplier for each shop: 25% off means 0.75, one third off means keeping $\frac{2}{3}$, and £28 off is a subtraction.
b. **Shop B** is cheapest at £60. The dearest is A at £67.50, so B is **£7.50** cheaper. Compare the three prices: £60 is the lowest and £67.50 the highest.
c. Total before discount $= 90 + 15 = 105 > 100$, so 40% off: $105 \times 0.60 =$ **£63** The

total of £105 is over £100, so the 40% discount applies to everything.

d. At A: $105 \times 0.75 =$ **£78.75**, so **shop C is cheaper (£63)**. At shop A both items are reduced by only 25%, so the total is higher than at shop C.

Answer to Question 16

a. T-shirt $12 \div 0.8 =$ **£15**; jeans $42 \div 0.7 =$ **£60**; hat $9 \div 0.75 =$ **£12**; bag $34 \div 0.85 =$ **£40**. Each sale price is a percentage of the original, so divide by the multiplier, for example $12 \div 0.8$ for 20% off.

b. Originals: $15 + 60 + 12 + 40 =$ **£127**. Sale: $12 + 42 + 9 + 34 =$ **£97**. Add the four original prices, then add the four sale prices.

c. Saving $= £30$. $30 \div 127 \times 100 =$ **23.6%** The saving is the difference between the totals; divide it by the original total and multiply by 100.

d. The mean of the four percentages is 22.5%, but the reductions apply to different amounts: bigger prices have more effect on the overall percentage. The jeans have the largest price and the largest reduction, so they pull the overall percentage above the simple mean.

Answer to Question 17

a. Simple interest: £50 every year, which is 5% of the original £1 000. Simple interest is always 5% of the starting amount, so it adds the same £50 every year.

b. Year 2: $1050 \times 1.05 =$ **£1 102.50**. Year 3: $1102.50 \times 1.05 =$ **£1 157.63**. Compound interest is 5% of the current total, so multiply by 1.05 each year and round to the nearest penny.

c. $1157.63 - 1150 =$ **£7.63** more. Subtract the simple interest total from the compound total after 3 years.

d. Year 5: $1000 \times 1.05^5 = 1276.28$. Year 6: 1340.10. Interest exceeds £300 for the first time after **6 years**. Keep multiplying by 1.05: after 5 years the interest is £276.28, which is less than £300.

Answer to Question 18

a. $2.10 - 1.20 =$ **£0.90** Profit = selling price − cost price.

b. $0.90 \div 1.20 \times 100 =$ **75%** Percentage profit is always worked out on the cost price, the amount the café paid.

c. New price $= 2.10 \times 1.1 =$ **£2.31**. Profit $= 1.11$, $1.11 \div 1.20 \times 100 =$ **92.5%** Multiply the old selling price by 1.1, then find the new profit as a percentage of the cost.

d. New cost $= 1.20 \times 1.05 =$ **£1.26**. Profit $= 2.31 - 1.26 =$ **£1.05**. Percentage $= 1.05 \div 1.26 \times 100 =$ **83.3%** The cost goes up by 5%, so multiply by 1.05; the profit per cake falls from £1.11 to £1.05.

e. $140 \times 1.05 =$ **£147** Multiply the profit per cake from part d) by the number of

cakes sold.

Answer to Question 19

a. Year 1: $600 \times 0.8 =$ **£480**. Year 2: $480 \times 0.85 =$ **£408**. Year 3: $408 \times 0.85 =$ **£346.80**. Use 0.8 for the first year and 0.85 for each of the next two years, always multiplying the latest value.
b. $600 - 346.80 =$ **£253.20** Subtract the value after 3 years from the original price.
c. $253.20 \div 600 \times 100 =$ **42.2%** Divide the loss by the original £600 and multiply by 100.
d. No. Each percentage is of a smaller amount, so the total is 42.2%, not 50%. The single multiplier is $0.8 \times 0.85 \times 0.85 = 0.578$, which is a 42.2% loss. Percentages cannot simply be added.

Answer to Question 20

a. $100 \times 1.2 = 120$; $120 \times 0.8 =$ **£96** Use the multipliers 1.2 and 0.8 in turn, starting from £100.
b. $1.2 \times 0.8 = 0.96$, so an overall decrease of **4%**. Multiply the two multipliers together; 0.96 means 96% is left.
c. Need multiplier $\frac{1}{1.25} = 0.8$, so a **20% decrease**. Divide 1 by 1.25 to find the multiplier that reverses the increase. Check: $125 \times 0.8 = 100$.
d. $1.1 \times 1.1 = 1.21$, so **21% increase**. Multiply the two multipliers: 1.21 means a 21% increase overall.
e. Let $x\% = r$ as a decimal. The multiplier is $(1 + r)(1 - r) = 1 - r^2$, which is less than 1 for any $r > 0$, so the price always falls. Expanding $(1 + r)(1 - r)$ gives a difference of two squares. Since r^2 is positive, the multiplier is always below 1.