Test answers - Number: approximation, error intervals and calculators

Year: Year 9 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. Two decimal places are decided by the digit in the thousandths place. In 5.0764 that digit is 6, which is 5 or more, so the 7 rounds up to 8: **5.08**. (1 mark)
b. The first significant figure is 5; the three significant figures are 5, 0 and 7. The zero counts because it lies between non-zero digits. The next digit, 6, rounds the 7 up: **5.08**. (1 mark)
c. Leading zeros are not significant. The first two significant figures are 7 and 3, and the next digit 8 rounds the 3 up to 4: **0.0074**. (1 mark)
d. The first significant figure is the 3 in the ten-thousands column. The second figure, 6, rounds it up to 4, and zeros keep the place value: **40 000**. (1 mark)

Answer to Question 2 (4 marks)

a. Round each number to 1 significant figure: 6.9 becomes 7, 4.1 becomes 4 and 0.52 becomes 0.5 (1 mark). The estimate is $\frac{7 \times 4}{0.5} = \frac{28}{0.5} =$**56**; dividing by 0.5 is the same as doubling (1 mark).
b. Work out the numerator first, $6.9 \times 4.1 = 28.29$, then divide: $28.29 \div 0.52 = 54.403\ldots$, which is **54.4** to 3 significant figures. (1 mark)
c. 56 is more than 54.4, so it is an **over-estimate**. The denominator was rounded down from 0.52 to 0.5, and dividing by a smaller number makes the answer bigger. (1 mark)

The estimate is within 3% of the true value, so it confirms the calculator answer is sensible. Brackets around the numerator matter when typing this into a calculator.

Answer to Question 3 (4 marks)

a. The measurement is to the nearest 0.1 cm, so the true length can be up to half of 0.1, which is 0.05 cm, either side of 8.6. The error interval is $8.55 \leq l < 8.65$. (1 mark for each bound)
b. The mass is to the nearest 10 g, so the bounds are 5 g either side: $445 \leq m < 455$. (1 mark)
c. To 1 significant figure, 3000 is rounded to the nearest 1 000, so the bounds are 500 either side: $2500 \leq n < 3500$. (1 mark)

The lower bound uses $\leq$ because a value of exactly 8.55 rounds up to 8.6. The upper bound uses a strict $<$ because 8.65 would round to 8.7, not 8.6.

Answer to Question 4 (4 marks)

a. The distance is to the nearest kilometre, so half a kilometre either side: $59.5 \leq d < 60.5$ (1 mark). The time is to the nearest 0.1 hour, so 0.05 hour either side: $2.45 \leq t < 2.55$ (1 mark).
b. To make speed = distance ÷ time as large as possible, divide the greatest distance by the least time: $60.5 \div 2.45 = 24.69\ldots$, so the upper bound is **24.7 km/h** (1 mark). For the smallest speed, divide the least distance by the greatest time: $59.5 \div 2.55 = 23.33\ldots$, so the lower bound is **23.3 km/h** (1 mark).

Her speed using the rounded values is $60 \div 2.5 = 24$ km/h, which sits between the bounds as expected.

Answer to Question 5 (4 marks)

a. An irrational number cannot be written as a fraction of two integers; its decimal never ends and never repeats. The irrational numbers are $\sqrt{7}$ (1 mark) and π (1 mark).
b. $0.5 = \frac{1}{2}$, a fraction with integer numerator and denominator, so 0.5 is rational. (1 mark)
c. 49 is a square number, so $\sqrt{49} = 7$. The integer 7 can be written as $\frac{7}{1}$, so $\sqrt{49}$ is rational, not irrational. (1 mark)

The fraction $\frac{4}{9} = 0.444\ldots$ is a recurring decimal and also rational. A calculator showing $\sqrt{7} = 2.6457513\ldots$ only displays a few digits, but the pattern never settles into a repeat. Square roots of numbers that are not perfect squares are always irrational.