# Answers - Number: approximation, error intervals and calculators

Year: Year 9

## Answer to Question 1

a. The next digit is 6, so round up: **3.5** The first decimal place is 4 and the digit after it is 6, so the 4 goes up to 5.
b. The next digit is 7, so round up: **3.47** Keep two decimal places, 3.46, and look at the third: 7 is 5 or more, so 3.46 becomes 3.47.
c. The next digit is 4, so round down: **12.05** Keep 12.05 and look at the next digit, 4, which is less than 5, so the number stays 12.05.
d. The next digit is 9, so round up, and $0.09 + 0.01 =$ **0.10** Keep 0.09 and look at the next digit, 9. Rounding up gives 0.10; the final zero must be written to show 2 dp.

## Answer to Question 2

a. 1 sf: the digit 4, next digit 8, round up: **50 000** The first significant figure is the 4 in the ten-thousands place, so the answer must keep its place value with zeros.
b. 2 sf: 48, next digit 7, round up: **49 000** Keep the first two digits 4 and 8; the 7 after them means the 8 rounds up to 9.
c. 3 sf: 487, next digit 2, round down: **48 700** Keep 4, 8 and 7; the next digit 2 is less than 5, so the 7 stays the same.
d. The first sf is 4; 2 sf is 45, next digit 8, so round up: **0.0046** The zeros at the start are not significant. The first two significant figures are 4 and 5, and the next digit 8 rounds 5 up to 6.

## Answer to Question 3

a. $5 \times 20 =$ **100** (exact $97.02$) Round 4.9 to 5 and 19.8 to 20. The estimate is very close to the exact value.
b. $300 \div 6 =$ **50** (exact $52.9$ to 3 sf) Round 312 to 300 and 5.9 to 6. The estimate is a little smaller than the exact value.
c. $\dfrac{8+12}{4} =$ **5** (exact $5.10$ to 3 sf) Round each number to 1 significant figure, then work out the top before dividing.
d. $\sqrt{49} =$ **7** (exact $7.07$) Since $7^2 = 49$, the square root of 50 is just over 7.

## Answer to Question 4

a. Half of 1 is 0.5: $6.5 \leq x$ The accuracy is 1, so go half a unit below and half a unit above 7.
b. Half of 0.1 is 0.05: $3.35 \leq x$ The accuracy is 0.1, so go 0.05 below and 0.05 above 3.4.
c. Half of 10 is 5: $245 \leq x$ The accuracy is 10, so go 5 below and 5 above 250.
d. The accuracy is 0.1, so half is 0.05: $0.75 \leq x$ The first significant figure 8 is in the tenths place, so the accuracy is 0.1 and the bounds are 0.05 either side.

## Answer to Question 5

a. Lower **11.5 cm**, upper **12.5 cm** The measurement is to the nearest 1 cm, so the bounds are 0.5 cm either side of 12 cm.
b. Lower **4.45 kg**, upper **4.55 kg** To the nearest 0.1 kg means the bounds are 0.05 kg either side of 4.5 kg.
c. Lower **9.75 s**, upper **9.85 s** To the nearest 0.1 s means the bounds are 0.05 s either side of 9.8 s.
d. Lower **195 ml**, upper **205 ml** To the nearest 10 ml means the bounds are 5 ml either side of 200 ml.

## Answer to Question 6

a. $\sqrt{16} = 4$, an integer, so **rational** 16 is a square number, so its square root is a whole number, which can be written as $\frac{4}{1}$.
b. **irrational** (the decimal never ends or repeats) 5 is not a square number, so its square root cannot be written as a fraction of two integers.
c. **rational** (a fraction) Any fraction of two integers is rational by definition, even though $\frac{22}{7}$ is close to $\pi$.
d. $0.2 = \frac{1}{5}$, **rational** A terminating decimal can always be written as a fraction.
e. **irrational** $\pi$ cannot be written exactly as a fraction; $\frac{22}{7}$ is only an approximation.
f. an integer, so **rational** Every integer is rational because it can be written as $\frac{-3}{1}$.

## Answer to Question 7

a. Nearest hundred: **38 500**. Nearest thousand: **38 000**. For the nearest hundred, the tens digit 6 rounds 38 400 up; for the nearest thousand, the hundreds digit 4 rounds down.
b. $38\,450 \leq n$ Half of 100 is 50, so the bounds are 50 below and 50 above 38 500.
c. $38\,450 \leq 38\,462$ is true. The true value is greater than the lower bound and less than the upper bound, so it fits the newspaper figure.

d. No: 38 560 is above 38 550, so it would round to 38 600. Numbers from 38 550 upwards round to 38 600, so 38 560 cannot be the true crowd.
e. $40\,000 \times 30 =$ **£1 200 000** Rounding to 1 significant figure gives an easy multiplication. The true amount is a little less, because both numbers were rounded up.

## Answer to Question 8

a. $3 + 4 + 1 + 6 =$ **£14** £2.95 rounds to £3, £4.10 to £4, £0.99 to £1 and £6.05 to £6.
b. $2.95 + 4.10 + 0.99 + 6.05 =$ **£14.09** Add the prices with the decimal points lined up. The exact total is £14.09.
c. The estimate is £14, which is lower than £14.09, so it is a very slight **under-estimate**. Some prices were rounded up and some down, but overall the estimate is 9p too low. Kofi still has enough money.
d. Estimate: $7 \times 0.50 =$ **£3.50**. Exact: $7 \times 0.49 =$ **£3.43**. Rounding 49p up to 50p gives an over-estimate of 7p, so the exact cost is slightly less than the estimate.

## Answer to Question 9

a. $300 \div 40 =$ **7.5** 312 rounds to 300 and 38.6 rounds to 40, which makes the division easy.
b. $312 \div 38.6 = 8.083\ldots =$ **8.08** to 3 sf Divide on a calculator and round to 3 significant figures, looking at the fourth figure 3.
c. $8.08 - 7.5 =$ **0.58** miles per litre Subtract the estimate from the exact value; the estimate is too small because 38.6 was rounded up.
d. $38.6 \times 1.45 = 55.97$, so **£55.97** Multiply on a calculator: $55.97$ exactly, so no further rounding is needed. The petrol costs £55.97.

## Answer to Question 10

a. Length: $4.55 \leq l$. Width: $3.75 \leq w$. Each measurement is to 1 dp, so the bounds are 0.05 m either side.
b. $4.55 \times 3.75 =$ **17.0625 m²** The smallest area uses the lower bound of both the length and the width.
c. $4.65 \times 3.85 =$ **17.9025 m²** The largest area uses the upper bound of both measurements.
d. The largest area is 17.9 m², so she should buy **18 m²**. She must cover even the largest possible floor area, so she rounds 17.9 up to 18, not down.
e. $18 \times 13.50 =$ **£243** Multiply the number of square metres by the price per square metre: the flooring costs £243.

## Answer to Question 11

a. $100 \div 10.4 = 9.615... =$ **9.62 m/s** Speed = distance ÷ time; the calculator shows 9.6153..., which rounds to 9.62.
b. Distance: $99.5 \leq d$. Time: $10.35 \leq t$. Distance is to the nearest metre (0.5 either side) and time to the nearest 0.1 s (0.05 either side).
c. Upper bound $= 100.5 \div 10.35 =$ **9.71 m/s** (to 2 dp) The greatest speed uses the longest distance and the shortest time.
d. Lower bound $= 99.5 \div 10.45 =$ **9.52 m/s** (to 2 dp) The least speed uses the shortest distance and the longest time.
e. Both bounds round to 9.5 and 9.7 at 2 sf, so they disagree at 2 sf; to 1 sf both are **10 m/s**. Her speed is about 10 m/s. A suitable accuracy is one where the upper and lower bounds round to the same value, so the answer can be trusted.

## Answer to Question 12

a. $11.05 \div 2.7 = 4.0925... =$ **4.09** Work out the top and the bottom separately, then divide. The fourth significant figure is 2, so round down.
b. $\sqrt{42.25 + 10.24} = \sqrt{52.49} =$ **7.24** Square both numbers, add them, then take the square root of the total.
c. $12.5 \div 6 =$ **2.0833...** Brackets are worked out first, so the calculator must add $2.5 + 3.5 = 6$ before dividing. Kamal's answer was wrong because he left out the brackets.
d. **2.08** to 2 dp The third decimal place is 3, so round down to 2.08.

## Answer to Question 13

a. $24\,550 \leq n$ The third significant figure is in the hundreds place, so the bounds are 50 either side of 24 600.
b. **24 550** The lower bound 24 550 is included in the interval, and it is a whole number.
c. **24 649** (the upper bound itself is not included) The interval stops just before 24 650, so the largest whole number inside it is 24 649.
d. $25\,000 \div 8 =$ **3 125 people** Rounding the population to 25 000 makes division by 8 easy; the answer is an estimate only.

## Answer to Question 14

a. $5 \times 3 =$ **15 m²** Round 5.2 to 5 and 2.8 to 3, then multiply.
b. $5.2 \times 2.8 =$ **14.56 m²** Multiply the exact lengths on a calculator: the area is 14.56 m².
c. $15 \div 11 \approx 1.4$ litres Divide the estimated area by the coverage of one litre.
d. $14.56 \div 11 = 1.32$, so she needs **2 tins** (whole tins). Paint is sold in whole tins, so

1.32 litres must be rounded up. She must buy 2 tins.

## Answer to Question 15

a. Row 1: **4678.3, 4700, 4700**. Row 2: **0.1, 0.054, 0**. Row 3: **12996.5, 13000, 13000**. For 1 dp look at the second decimal place, for 2 sf keep the first two non-zero digits, and for the nearest 100 look at the tens digit.
b. 2 dp: **0.05**. 2 sf: **0.054**. The zero after the decimal point is not significant. Decimal places count from the decimal point, but significant figures start at the first non-zero digit.
c. Rounding to 2 sf keeps only 1 and 2, then rounds up because the next digit is 9, so 12 996.5 becomes 13 000. The digits 2, 9 and 9 all round up together, which changes several place values at once.
d. $12\,500 \leq x$ (half of 1000 is 500). To 2 sf, 13 000 is accurate to the nearest 1000, so the bounds are 500 either side.

## Answer to Question 16

a. Lower $= 8.35 \times 5.15 =$ **43.0025 cm²**. Upper $= 8.45 \times 5.25 =$ **44.3625 cm²**. The smallest area uses both lower bounds and the largest area uses both upper bounds.
b. Lower $= 2(8.35 + 5.15) =$ **27 cm**. Upper $= 2(8.45 + 5.25) =$ **27.4 cm**. The perimeter is twice the sum of the length and the width; use the bounds in the same way.
c. 43.68 uses more accuracy than the data allow. The true area is anywhere from 43.0 to 44.4. The measurements are only accurate to 1 dp, so the area cannot be known to 2 decimal places.
d. To 2 sf both bounds round to 43 and 44, so they disagree. To 1 sf both give **40 cm²**, so the area is about 40 cm². Give the answer to the accuracy at which both bounds agree.

## Answer to Question 17

a. Mass: $245 \leq m$. Volume: $79.5 \leq V$. The mass is to the nearest 10 g (5 g either side) and the volume to the nearest 1 cm³ (0.5 cm³ either side).
b. Greatest $= 255 \div 79.5 =$ **3.2075 g/cm³** To make the density as large as possible, divide the largest mass by the smallest volume.
c. Least $= 245 \div 80.5 =$ **3.0435 g/cm³** To make the density as small as possible, divide the smallest mass by the largest volume.
d. Both are 3 to 1 sf, so the density is **3 g/cm³** to 1 sf. Both bounds round to 3, so 3 g/cm³ is a reliable answer to 1 significant figure.
e. For the greatest answer you divide the biggest top by the smallest bottom: upper bound of mass over lower bound of volume. Dividing by a smaller number gives a

bigger answer, which is why the lower bound of the volume is used.

## Answer to Question 18

a. $42.5 \leq a$; $0.615 \leq b$; $1150 \leq c$; $2.95 \leq d$. In each case go half of the rounding unit below and above the value. For $c$, 2 significant figures means the nearest 100.
b. Widths: $a$ is 5, $b$ is 0.01, $c$ is 100, $d$ is 0.1. Subtract the lower bound from the upper bound; the width always equals the rounding unit.
c. $c$ has the largest width (100), so it is the least certain in absolute terms. The width of 100 is bigger than all the others, so $c$ is the least precisely known quantity.
d. $c = 1249$ is possible. $c = 1250$ is **not** possible because the upper bound is not included. Any number from 1150 up to, but not including, 1250 rounds to 1200; 1250 itself rounds up to 1300.

## Answer to Question 19

a. $\frac{1}{3}$, $\frac{1}{8}$ and $\frac{1}{7}$ are rational. Their decimals either **stop** or **repeat** in a pattern. A rational number can be written as a fraction of two integers, and its decimal stops or repeats a block of digits.
b. Their decimals go on forever with no repeating pattern, so they cannot be written as fractions. Mathematicians have proved that no fraction is exactly equal to $\sqrt{2}$ or to $\pi$.
c. $2 + \sqrt{3}$ is **irrational** (an integer plus an irrational number). $\sqrt{2} \times \sqrt{2} = 2$, which is **rational**. Adding an integer to an irrational number cannot remove the endless non-repeating decimal, but squaring a square root gives back the original whole number.
d. $10x - x = 7.777\ldots - 0.777\ldots = 7$, so $9x = 7$ and $x = \frac{7}{9}$. Subtracting removes the endless repeating part, which leaves a simple equation. Check: $7 \div 9 = 0.777\ldots$.

## Answer to Question 20

a. Area: $23.5 \leq A$. Length: $5.5 \leq L$. Both measurements are to the nearest whole unit, so the bounds are 0.5 either side.
b. Greatest width $= 24.5 \div 5.5 =$ **4.4545... cm** Width = area ÷ length; the largest width needs the largest area and the smallest length.
c. Least width $= 23.5 \div 6.5 =$ **3.6154... cm** The least width needs the smallest area divided by the largest length.
d. Yes: $3.6154$. 4 cm lies between the least and the greatest possible widths, so it is possible.
e. No: $4.5 > 4.4545$, which is the greatest possible width. 4.5 cm is bigger than the greatest possible width, so it is not possible.