Lesson - Graphs: linear and quadratic

Year: Year 9

This lesson covers drawing graphs from tables of values, linear graphs, quadratic graphs, reading values from graphs and approximate solutions of simultaneous equations.

Starter activity

A taxi charges £3 to start and then £2 for each mile. Fill in a table for 0, 1, 2, 3 and 4 miles and plot the points on squared paper. What do you notice about the points? Now throw a ball and record how high it is every half-second, or imagine its path. Is its path a straight line? What shape do you think it makes?

Drawing graphs from equations

Method: drawing a graph from an equation

1. Make a **table of values** for a sensible range of x.
2. Work out y for each x.
3. Plot the points (x, y) on a grid with equal steps along each axis (1, 2 or 5 per square).
4. Join the points: a ruler for lines, a smooth curve for quadratics.

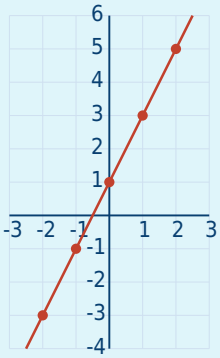
Linear graphs: y = mx + c

Definition: gradient and y-intercept

The graph of a linear function is a **straight line**. In $y = mx + c$, m is the **gradient**: how many units y goes up when x goes up by 1. c is the **y-intercept**: where the line crosses the y-axis, at the point $(0, c)$.

Worked example 1: drawing a line

Draw $y = 2x + 1$ for $-2 \leq x \leq 2$.



x	-2	-1	0	1	2
y	-3	-1	1	3	5

Plot $(-2,-3)$, $(-1,-1)$, $(0,1)$, $(1,3)$, $(2,5)$ and join them with a ruler. The gradient is 2 and the y-intercept is 1.

Worked example 2: finding an equation

Find the equation of the line through $(1,2)$ and $(3,8)$.

Gradient $= \dfrac{8-2}{3-1} = 3$. So $y = 3x + c$. Substitute $(1,2)$: $2 = 3 + c$, $c = -1$. The equation is $y = 3x - 1$.

Key fact: points, parallel lines and intercepts

- A point lies on a line if its coordinates satisfy the equation. $(2,7)$ is on $y = 3x + 1$, because $3 \times 2 + 1 = 7$. $(4,10)$ is not on $y = 2x + 3$, because $2 \times 4 + 3 = 11$.
- **Parallel lines** have the same gradient.
- For the y-intercept set $x = 0$; for the x-intercept set $y = 0$. For $y = 2x - 6$: the y-intercept is -6 and $0 = 2x - 6$ gives $x = 3$.

Quadratic graphs

Definition: quadratic function and parabola

A **quadratic function** has an x^2 term as its highest power, such as $y = x^2 - 4$. Its graph is a smooth, symmetrical, U-shaped curve called a **parabola**. The lowest point is the **turning point** (vertex). If the x^2 term is negative, as in $y = -x^2$, the curve is upside down. The points where $y = 0$ are the **roots**.

Worked example 3: $y = x^2 - 4$

Draw $y = x^2 - 4$ and find the roots, turning point and line of symmetry.

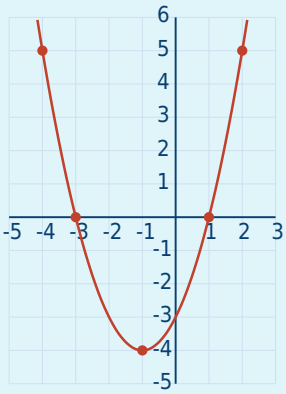
x	-3	-2	-1	0	1	2	3
$y = x^2 - 4$	5	0	-3	-4	-3	0	5

The graph crosses the x-axis at $x = -2$ and $x = 2$ (the roots). The turning point is $(0, -4)$ and the line of symmetry is $x = 0$. Join the points with a smooth curve, not straight segments, and keep it rounded at the bottom.

Worked example 4: $y = x^2 + 2x - 3$

Find the roots, turning point and line of symmetry of $y = x^2 + 2x - 3$.

For $x = -4$ to 2 the table gives $y = 5, 0, -3, -4, -3, 0, 5$. The roots are $x = -3$ and $x = 1$. The turning point is $(-1, -4)$ and the line of symmetry is $x = -1$, halfway between the roots.



Watch out: negative numbers and curves

$(-3)^2 = 9$, not -9. Never join a quadratic with straight lines or make a pointed bottom: equal heights on both sides of the turning point tell you the table is right.

Reading values from graphs

Worked example 5: reading values

Find y when $x = 1.5$ on $y = x^2 - 4$, and solve $x^2 = 5$ from the graph of $y = x^2$.

$1.5^2 - 4 = -1.75$. To solve $x^2 = 5$, draw the line $y = 5$ on the graph of $y = x^2$; it meets the curve at about $x = 2.2$ and $x = -2.2$. A quadratic equation can have two, one or no solutions.

Approximate solutions of simultaneous equations

Rule: solving simultaneously by graphs

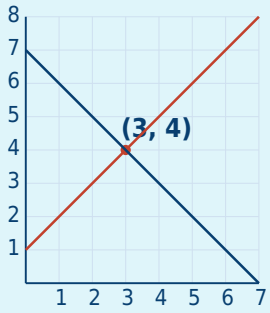
Two equations are solved **simultaneously** when we find the x and y that satisfy both.

Draw the graphs of both equations: the point where they cross gives the solution.

Worked example 6: two lines

Solve $y = x + 1$ and $y = 7 - x$.

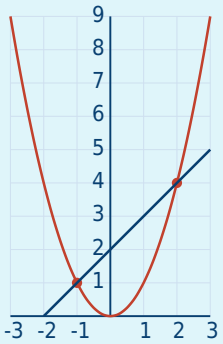
The lines cross at $(3, 4)$. Check: $3 + 1 = 4$ and $7 - 3 = 4$. So $x = 3$, $y = 4$.



Worked example 7: a line and a curve

Solve $x^2 = x + 2$ by drawing graphs.

Draw $y = x^2$ and $y = x + 2$. They cross at $(-1, 1)$ and $(2, 4)$, so $x = -1$ or $x = 2$.



Watch out: common mistakes

- Joining a quadratic curve with straight lines or making a pointed bottom.
- Mistakes with negative numbers: $(-3)^2 = 9$, not -9.
- Forgetting that the y-intercept is the value of c in $y = mx + c$, not the gradient.
- Reading only one solution from a quadratic when the graph crosses twice.
- Poorly chosen scales, which make the graph hard to read.

Worked example 8: finding the equation of a line from a graph

A line passes through $(2, 1)$ and $(6, 9)$. Find its equation.

Gradient $m = \dfrac{9 - 1}{6 - 2} = \dfrac{8}{4} = 2$. Substitute $(2, 1)$ into $y = 2x + c$: $1 = 4 + c$, so $c = -3$.

The equation is $y = 2x - 3$. Check with the other point: $2 \times 6 - 3 = 9$, as required.

Worked example 9: rearranging into the form y = mx + c

Find the gradient and intercept of $3y - 6x = 12$.

Add $6x$ to both sides: $3y = 6x + 12$. Divide every term by 3: $y = 2x + 4$. The gradient is 2 and the y-intercept is 4.

Worked example 10: the turning point of a quadratic

Find the turning point of $y = x^2 - 4x + 1$ from a table.

For $x = 0, 1, 2, 3, 4$ the values of y are $1, -2, -3, -2, 1$. The lowest value is -3 at $x = 2$. The turning point is $(2, -3)$ and the line of symmetry is $x = 2$. Notice that the values on both sides of $x = 2$ match, which is a good check for your table.

Note: choosing a scale

When you draw a graph, choose scales that use most of the page. Count how far x and y need to go from your table. Mark the axes at equal intervals, label them, and join the points of a quadratic with a smooth curve drawn in one movement. Never use a ruler for a curve and never join the points with short straight lines.

Watch out: negative numbers in tables

- $(-3)^2 = 9$, not -9. Use brackets in your calculator: $(-3)^2$.
- In $y = x^2 + 2x - 3$, work out x^2, then $2x$, then add and subtract: for $x = -2$ it is $4 - 4 - 3 = -3$.
- A parabola has no sharp corners and it is symmetrical: if your table is not symmetrical about the turning point, recheck it.

Did you know?

The path of a thrown ball is a parabola. Galileo worked this out in the 1600s by rolling balls down slopes. Today, engineers use parabolas for car headlight reflectors and satellite dishes, because a parabola collects all parallel rays at a single point.

Linear or quadratic?

Key fact: telling the graphs apart

You can often recognise the type of graph from its equation before you draw it. If the highest power of x is x itself, the graph is a straight line. If the equation contains x^2 and no higher power, the graph is a parabola. A positive x^2 term gives a U-shaped curve with a minimum point; a negative x^2 term, as in $y = 6x - x^2$, gives an upside-down curve with a maximum point. In a table of values, the differences between consecutive y-values are constant for a line, but for a quadratic it is the second

differences that are constant.

Worked example 11: an upside-down parabola

Complete a table for $y = 4x - x^2$ for $x = 0$ to 4 and describe the graph.

Work out each value carefully, squaring x before you subtract. When $x = 0$, $y = 0$. When $x = 1$, $y = 4 - 1 = 3$. When $x = 2$, $y = 8 - 4 = 4$. When $x = 3$, $y = 12 - 9 = 3$. When $x = 4$, $y = 16 - 16 = 0$. The values rise and then fall, so the curve is an upside-down U. Its highest point, the maximum, is $(2, 4)$, and its line of symmetry is $x = 2$. The curve crosses the x-axis at $x = 0$ and $x = 4$, which are the roots of $4x - x^2 = 0$.

Method: plotting a quadratic accurately

1. Make a table of values, using brackets around negative numbers so that $(-3)^2$ is worked out as $+9$.
2. Choose a scale for each axis that fits the smallest and largest values in your table.
3. Plot every point with a small, neat cross.
4. Join the points with one smooth curve drawn freehand. Never join them with a ruler: a parabola has no straight parts.
5. Check the shape is symmetrical. If one point does not fit the curve, recalculate that value before you draw through it.

Watch out: reading from a graph

Answers read from a graph are estimates, so give them to the accuracy of the scale, for example to the nearest 0.1 or 0.5. When you solve an equation such as $x^2 - 4 = 3$ with a graph, draw the horizontal line $y = 3$ and read the x-values where it crosses the curve: there are usually two answers, one on each side of the line of symmetry. Forgetting the second answer is a very common way to lose a mark. If the horizontal line only touches the curve at the turning point, there is just one answer, and if it misses the curve completely, the equation has no solutions at all.

Otto's tip

Include the turning point in your table and check the symmetry. Equal heights on both sides tell you the table is right. Hoot hoot!