Answers - Graphs: linear and quadratic

Year: Year 9

Answer to Question 1

a. $x = -2$: $2(-2) + 1 = -3$; $x = -1$: -1; $x = 0$: 1; $x = 1$: 3; $x = 2$: 5; $x = 3$: 7. Row: $-3, -1, 1, 3, 5, 7$. Substitute each value of x into $2x + 1$, using brackets for the negative numbers.

b. $(0, 1)$, where $x = 0$. The line crosses the y-axis where $x = 0$. The table shows $y = 1$ there, so the point is $(0, 1)$.

c. The gradient is **2**: y goes up by 2 each time x goes up by 1. Each y-value is 2 more than the one before, because the coefficient of x is 2.

d. $2 \times 10 + 1 =$ **21** Substitute $x = 10$ into the equation: $y = 21$.

Answer to Question 2

a. $m =$ **3**, $c = -4$ The equation is already in the form $y = mx + c$: the number in front of x is the gradient and the number on its own is the intercept.

b. $y = -1x + 5$, so $m = -1$, $c =$ **5** The term $-x$ means $-1 \times x$, so the gradient is negative and the line slopes downwards.

c. $m = \frac{1}{2}$, $c =$ **2** A fractional gradient means the line rises 1 unit for every 2 units across.

d. Divide by 2: $y = 2x + 3$, so $m =$ **2**, $c =$ **3** The equation must start $y =$ before you can read m and c, so divide every term by 2 first.

Answer to Question 3

a. $m = (7 - 3) \div 2 = 2$; $c = 3$: $y = 2x + 3$ The point $(0, 3)$ is on the y-axis, so the intercept is 3. Check with $(2, 7)$: $2 \times 2 + 3 = 7$.

b. $m = (0 - 5) \div 5 = -1$; $c = 5$: $y = -x + 5$ The line goes down 5 for every 5 across, so the gradient is -1. Check: $-5 + 5 = 0$.

c. $m = (7 - 3) \div (3 - 1) = 2$. Using $(1, 3)$: $3 = 2 + c$, so $c = 1$: $y = 2x + 1$ Neither point is on the y-axis, so substitute one point into $y = 2x + c$ to find c. Check with $(3, 7)$: $6 + 1 = 7$.

d. $m = (7 + 1) \div 4 = 2$; $c = -1$: $y = 2x - 1$ The change in y is $7 - (-1) = 8$ and the change in x is 4. Check with $(4, 7)$: $8 - 1 = 7$.

Answer to Question 4

a. $3(4) - 2 = 10$. **Yes.** Substitute $x = 4$: the result equals the y-coordinate 10, so the

point is on the line.

b. $3(-1) - 2 = -5$. **Yes.** Substitute $x = -1$: the result equals the y-coordinate -5, so the point is on the line.
c. $3(2) - 2 = 4$, not 5. **No.** The value of y given by the equation is 4, but the point has $y = 5$, so it lies just above the line.
d. Parallel lines have the same gradient 3. $y = 3x + 7$ has $m = 3$ (**parallel**). $2y = 6x + 1$ gives $y = 3x + 0.5$ (**parallel**). The others have gradients -3 and 2. Always rearrange into $y = mx + c$ before comparing gradients. Two lines are parallel: $y = 3x + 7$ and $2y = 6x + 1$.

Answer to Question 5

a. $(-3)^2 - 4 = 5$; 0; -3; -4; -3; 0; 5. Row: $5, 0, -3, -4, -3, 0, 5$. Square x first and then subtract 4; a negative number squared is positive, so $(-3)^2 = 9$.
b. $(0, -4)$ The smallest value in the table is -4, when $x = 0$, so the turning point is a minimum at $(0, -4)$.
c. $y = 0$ at $x = -2$ **and** $x = 2$. The curve crosses the x-axis where $y = 0$; the table shows this at two values of x.
d. $x = 0$ (the y-axis). The table is symmetrical about $x = 0$: for example $x = -2$ and $x = 2$ both give $y = 0$.

Answer to Question 6

a. $(-2)^2 + 2(-2) - 3 = 4 - 4 - 3 = -3$. Matches. Substituting into the equation gives the same value as the table, so the table entry is correct.
b. $y = 0$ at $x = -3$ **and** $x = 1$. The roots are the solutions of $x^2 + 2x - 3 = 0$, which are the x-values where the curve meets the x-axis.
c. The minimum is -4, at $x = -1$. The values decrease and then increase, and the smallest one is -4 at $x = -1$, so this is the turning point.
d. $y = 5$ at $x = -4$ **and** $x = 2$. Look for the value 5 in the y-row: it appears at two values of x, one on each side of the line of symmetry.

Answer to Question 7

a. $3, 7, 11, 15, 19$ (since $2x + 3$ for $x = 0, 2, 4, 6, 8$). Substitute each number of miles into $2x + 3$. Each extra 2 miles adds £4 to the fare.
b. The gradient is the cost per mile (£2) and the intercept is the fixed fee (£3). The intercept is the fare when $x = 0$, before any miles are travelled; the gradient is the extra cost for each mile.
c. $2 \times 7 + 3 =$ **£17** Substitute $x = 7$ into the equation. A 7-mile journey costs £17.
d. $2x + 3 = 25$, so $2x = 22$ and $x =$ **11 miles**. Set the fare equal to 25 and solve the equation. Sunita can travel 11 miles.
e. x is always true, so the £1-per-mile firm is cheaper at **every** distance. The second

firm's line $y = x$ starts lower and rises more slowly, so it is always below $y = 2x + 3$ for positive distances.

Answer to Question 8

a. $15 \times 6 + 20 =$ **£110** Substitute $x = 6$ into the equation. Six months cost £110.
b. $15x + 20 = 200$, so $15x = 180$ and $x =$ **12 months**. Set the cost equal to 200, subtract 20 and divide by 15. It takes 12 months.
c. It is a straight line with gradient 15 that crosses the y-axis at $(0, 20)$: the joining fee. The y-intercept is the cost when $x = 0$, so it is the joining fee of £20.
d. $y = 25x$. Equal when $25x = 15x + 20$, so $10x = 20$ and $x =$ **2 months** (£50 each). The second gym's line goes through the origin because there is no joining fee. After 2 months both gyms cost £50; after that the first gym is cheaper.

Answer to Question 9

a. $20 - 2.5 \times 3 = 20 - 7.5 =$ **12.5 cm** Substitute $x = 3$ into the formula. After 3 hours the candle is 12.5 cm tall.
b. The candle burns down by **2.5 cm every hour** (negative because the height decreases). The gradient is the change in height per hour; it is negative because the candle gets shorter.
c. $20 - 2.5x = 0$, so $x = 20 \div 2.5 =$ **8 hours**. The candle has burnt out when its height is 0, so set the formula equal to 0 and solve. It lasts 8 hours.
d. The end points are $(0, 20)$ and $(8, 0)$. The line starts at the original height of 20 cm and finishes on the x-axis when the candle has burnt out.

Answer to Question 10

a. A: $5 + 6 =$ **11 cm**. B: $8 + 6 =$ **14 cm**. Substitute $x = 4$ into both equations. After 4 weeks plant A is 11 cm tall and plant B is 14 cm tall.
b. Yes, both have gradient 1.5, so both grow 1.5 cm a week and B stays 3 cm taller. Lines with the same gradient are parallel, so the difference in height of 3 cm never changes.
c. $5 + 1.5x = 29$, so $1.5x = 24$ and $x =$ **16 weeks**. Set plant A's height equal to 29 and solve the equation. This happens after 16 weeks.
d. $2x + 5 = 1.5x + 5$ gives $0.5x = 0$, so $x = 0$: they start at the same height and C grows faster, so they are equal **only at the start**. The two equations give the same height only when $x = 0$; after that plant C grows 0.5 cm a week faster.

Answer to Question 11

a. $0, 5, 8, 9, 8, 5, 0$ Substitute each value of x into $x(6 - x)$. For example, when $x = 2$,

$h = 2 \times 4 = 8$.

b. The greatest height is **9 m** at $x = 3$ **m**. The largest value in the table is 9, in the middle of the table at $x = 3$, which is the turning point.
c. $h = 5$ at $x = 1$ **and** $x = 5$. The value 5 appears twice in the table, at the same distance on either side of $x = 3$.
d. It lands where $h = 0$ again, at **6 m**. The ball starts at height 0 when $x = 0$ and returns to height 0 when $x = 6$, so it lands 6 m away.

Answer to Question 12

a. $20^2 \div 20 = 20$; $30^2 \div 20 = 45$; $40^2 \div 20 = 80$; $60^2 \div 20 = 180$. Square the speed first, then divide by 20. For example, $30^2 = 900$ and $900 \div 20 = 45$.
b. No. Doubling from 20 to 40 mph multiplies the distance by $80 \div 20 = 4$. Doubling the speed multiplies the braking distance by 4, because the speed is squared in the formula.
c. $v^2 = 125 \times 20 = 2500$, so $v =$ **50 mph**. Multiply both sides by 20 and then take the positive square root. The speed is 50 mph.
d. No. The relation involves v^2, so the graph is a curve (a parabola) that gets steeper. Equal increases in speed give bigger and bigger increases in distance, so the points do not lie on a straight line.

Answer to Question 13

a. A: $0.05 \times 200 + 10 = 20$. B: $0.1 \times 200 = 20$. Substitute $x = 200$ into each equation. Both plans cost £20 for 200 minutes.
b. The lines cross at **(200, 20)**: 200 minutes costs £20 on both plans. The lines cross where the costs are equal; the tables show this happens at 200 minutes.
c. At 150 min: A $= 7.5 + 10 = £17.50$, B $= £15$, so **Plan B** is cheaper. Substitute $x = 150$ into each equation and compare. Plan B is £2.50 cheaper for 150 minutes.
d. At 300 min A costs £25 and B costs £30, so **Plan A** is cheaper. Algebra: $0.05x + 10 = 0.1x$ gives $10 = 0.05x$ and $x = 200$. For more than 200 minutes Plan A is cheaper because its cost per minute is lower. The algebra confirms the crossing at 200 minutes.

Answer to Question 14

a. $\frac{1}{2}(6) + 2 = 5$. **Yes.** Substitute $x = 6$ into the equation of the fence; the result equals the y-coordinate, so the tree is on the fence line.
b. $\frac{1}{2}(4) + 2 = 4$, not 3. **No.** The equation gives $y = 4$ at $x = 4$, but the pond has $y = 3$, so the pond is 1 m below the line.
c. $0 = \frac{1}{2}x + 2$, so $x = -4$: at $(-4, 0)$. The x-axis is where $y = 0$. Subtract 2 and multiply by 2 to solve the equation.

d. Parallel means the same gradient $\frac{1}{2}$ and $c = -1$: $y = \frac{1}{2}x - 1$. Parallel lines have the same gradient, and the point $(0, -1)$ gives the intercept.

Answer to Question 15

a. Line 1: 2, 4, x-int -2. Line 2: -3, 6, x-int 2. Line 3: $\frac{1}{2}$, -1, x-int 2. Line 4: 4, 0, x-int 0. Read m and c from each equation; for the x-intercept set $y = 0$ and solve the equation.
b. $y = 4x$, since $c = 0$. A line passes through the origin when its y-intercept is 0, that is when $c = 0$.
c. Lines 2 and 3 both cross at $x = 2$. Lines 2 and 3 both have an x-intercept of 2, so they cross each other on the x-axis at $(2, 0)$.
d. Through $(2, 0)$ with $m = 2$: $0 = 4 + c$, so $c = -4$ and $y = 2x - 4$. Substitute the point $(2, 0)$ into $y = 2x + c$ to find c. Check: $2 \times 2 - 4 = 0$.

Answer to Question 16

a. Roots $x = -3$ **and** $x = 1$. Turning point $(-1, -4)$. The roots are where the curve meets the x-axis and the turning point is the lowest point of the U.
b. Line of symmetry $x = -1$. Mean of roots: $(-3 + 1) \div 2 = -1$, the same. The line of symmetry passes through the turning point. It is always halfway between the two roots.
c. $y = -3$ at $x = -2$ **and** $x = 0$. Draw the horizontal line $y = -3$ and read the two x-values where it crosses the curve.
d. $0.25 + 1 - 3 = -1.75$, close to -1.8 on the graph. Substituting $x = 0.5$ gives -1.75, so the graph reading is a good estimate.
e. The x^2 term makes y increase faster as x moves away from the turning point, so the gradient keeps changing. In a linear graph the gradient is the same everywhere; in a quadratic the gradient changes all the time, which bends the graph into a curve.

Answer to Question 17

a. They cross at $(5, 15)$: after 5 weeks both have £15. The point where two graphs cross is where both people have the same amount of money at the same time.
b. $2x + 5 = 3x$ gives $x = 5$ and $y = 15$. Set the two expressions equal and solve; substitute $x = 5$ back to find $y = 15$.
c. Hannah $= 2 \times 3 + 5 = 11$; Tom $= 9$. **Hannah** has more. Substitute $x = 3$ into both equations: Hannah has £11 and Tom has £9, so Hannah has £2 more.
d. Hannah £21, Tom £24, so **Tom** has **£3** more. Use the table at 8 weeks. After 5 weeks Tom's faster saving puts him ahead.

Answer to Question 18

a. The two rows are equal at $x = -1$ (both 1) and at $x = 2$ (both 4). The line and the curve meet where the two rows of the table give the same y-value.
b. $(-1, 1)$ **and** $(2, 4)$. Each point of intersection uses the x-value from part a) and the common y-value.
c. $(-1)^2 = 1 = -1 + 2$ and $2^2 = 4 = 2 + 2$. Substituting each x-value gives the same number on both sides of the equation, so these are the solutions.
d. The curve and line meet where $x^2 = x + 2$. Rearranging gives $x^2 - x - 2 = 0$, so its solutions are the x-values where they cross. Subtracting $x + 2$ from both sides of $x^2 = x + 2$ gives the quadratic equation, so it has the same solutions.

Answer to Question 19

a. $8, 3, 0, -1, 0, 3, 8$ Substitute each value of x into $x^2 - 6x + 8$. For example, when $x = 3$, $y = 9 - 18 + 8 = -1$.
b. The lowest point is $(3, -1)$: 1 m below ground. The smallest value in the table is -1 at $x = 3$, so the bottom of the ramp is 1 m below the ground.
c. Between $x = 2$ **and** $x = 4$ the height is below 0. The height is negative only between the two roots $x = 2$ and $x = 4$.
d. $x = 3$: for example the values at $x = 2$ and $x = 4$ are both 0. Points at the same distance on either side of $x = 3$ have the same height, for example $x = 1$ and $x = 5$ both give 3.
e. The graph is symmetrical about $x = 3$, so the left half is the reflection of the right half. A parabola is symmetrical about the vertical line through its turning point, so the two halves are reflections of each other.

Answer to Question 20

a. $m = (11 - 5) \div (4 - 1) =$ **2** Use change in y divided by change in x between the two points.
b. $5 = 2(1) + c$, so $c = 3$: $y = 2x + 3$. Substitute one of the points into $y = 2x + c$ to find c. Check with $(4, 11)$: $8 + 3 = 11$.
c. $x = -2, -1, 0, 1, 2, 3$ gives $y = -1, 1, 3, 5, 7, 9$. Substitute each value of x into $y = 2x + 3$; the values go up by 2 each time.
d. For $y = x^2$: $4, 1, 0, 1, 4, 9$. They agree at $x = -1$ (both 1) and $x = 3$ (both 9). Points: $(-1, 1)$ **and** $(3, 9)$. Compare the two tables: the line and curve meet where the values are equal.
e. $x^2 = 2x + 3$ gives $x^2 - 2x - 3 = 0$.
$(x - 3)(x + 1) = x^2 + x - 3x - 3 = x^2 - 2x - 3$. So $x = 3$ or $x = -1$. Expanding the brackets gives back the quadratic, so its solutions $x = 3$ and $x = -1$ agree with the points found from the tables.