Test answers - Expanding binomials and taking out common factors

Year: Year 9 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. Multiply both terms by 5: $5 \times 2x = 10x$ and $5 \times (-3) = -15$. The answer is $10x - 15$. (1 mark)
b. Multiply each term in the first bracket by each term in the second: $x \times x = x^2$, $x \times (-3) = -3x$, $7 \times x = 7x$, $7 \times (-3) = -21$ (1 mark for the four products). Collect the middle terms, $-3x + 7x = 4x$: $x^2 + 4x - 21$ (1 mark).
c. The minus sign before the second bracket means multiply it by -1: $3x + 6 - x + 4$. Collect: $2x + 10$. (1 mark)

Check (b) with $x = 1$: $(1 + 7)(1 - 3) = 8 \times (-2) = -16$, and $1 + 4 - 21 = -16$. A grid method helps to avoid missing a product.

Answer to Question 2 (4 marks)

a. The HCF of 12 and 18 is 6, so take 6 outside: $12x \div 6 = 2x$ and $18 \div 6 = 3$. The answer is $6(2x - 3)$. (1 mark)
b. Both terms contain x: $x^2 = x \times x$ and $9x = 9 \times x$. Take x outside the bracket: $x(x + 9)$. (1 mark)
c. The HCF of 15 and 10 is 5, and both terms contain a, so the HCF is $5a$ (1 mark). Dividing: $15ab \div 5a = 3b$ and $10a^2 \div 5a = 2a$. The answer is $5a(3b + 2a)$, also written $5a(2a + 3b)$ (1 mark).

A partial factorisation such as $5(3ab + 2a^2)$ earns only 1 mark in (c). Always check that nothing more can come out of the bracket; then expand to check.

Answer to Question 3 (4 marks)

a. Write the square as two brackets: $(x + 5)(x + 5) = x^2 + 5x + 5x + 25$ (1 mark). The two middle terms add to $10x$: $x^2 + 10x + 25$ (1 mark). No marks are given for $x^2 + 25$, which forgets the middle terms.
b. Expand: $4x^2 - 6x + 6x - 9$. The middle terms cancel, leaving $4x^2 - 9$. This is a difference of two squares, $(2x)^2 - 3^2$. (1 mark)
c. Since $64 = 8^2$, use $a^2 - b^2 = (a + b)(a - b)$: $(x + 8)(x - 8)$. (1 mark)

Check (a) with $x = 1$: $6^2 = 36$ and $1 + 10 + 25 = 36$. Check (c) by expanding: $x^2 - 8x + 8x - 64 = x^2 - 64$.

Answer to Question 4 (4 marks)

a. Area = length × width $= (x+5)(x+3)$. Expanding gives $x^2+3x+5x+15$ (1 mark). The middle terms add to $8x$, so the area is $x^2+8x+15$, as required (1 mark).
b. Substitute $x=6$: $6^2+8\times 6+15=36+48+15=$**99 m²**. A quicker check uses the sides: $6+5=11$ m and $6+3=9$ m, and $11\times 9=99$. (1 mark)
c. Multiply the area by the price per square metre: $99\times 2.50=$**£247.50**. (1 mark)

In a "show that" question every step must be written, because the answer is already given.

Answer to Question 5 (4 marks)

a. Follow the steps in order. Adding 3 gives $n+3$; multiplying by 4 gives $4(n+3)$ (1 mark); subtracting 12 gives $4(n+3)-12$; dividing by the starting number gives $\dfrac{4(n+3)-12}{n}$ (1 mark).
b. Expand the numerator: $4n+12-12=4n$, because the 12s cancel (1 mark). Then $\dfrac{4n}{n}=4$, since n divides exactly when $n\neq 0$ (1 mark). The result is 4 for every allowed starting number, so Sam is right.

Testing with $n=7$: $7+3=10$, $10\times 4=40$, $40-12=28$, $28\div 7=4$. A numerical example shows the trick working once, but only the algebra proves it always works. The condition $n\neq 0$ matters because dividing by zero is undefined.