Answers - Expanding binomials and taking out common factors

Year: Year 9

Answer to Question 1

a. $3 \times x + 3 \times 5 = 3x + 15$ Each of the two terms in the bracket is multiplied by 3, so the answer is $3x + 15$.
b. $4 \times 2y - 4 \times 3 = 8y - 12$ Both terms are multiplied by 4; keep the minus sign with the 12, giving $8y - 12$.
c. $-2 \times a + (-2) \times 6 = -2a - 12$ The negative number outside multiplies both terms, and a negative times a positive is negative, so the answer is $-2a - 12$.
d. $x \times x + x \times 7 = x^2 + 7x$ Multiplying x by x gives x^2 and x by 7 gives $7x$, so the answer is $x^2 + 7x$.

Answer to Question 2

a. $3x + 6 + 2x + 10 = 5x + 16$ Collect the x terms $3x + 2x = 5x$ and the numbers $6 + 10 = 16$.
b. $10a - 5 - 3a - 12 = 7a - 17$ Collect $10a - 3a = 7a$ and $-5 - 12 = -17$; the second bracket is multiplied by -3, so both of its terms become negative.
c. $4y + 12 - y + 2 = 3y + 14$ (the minus changes both signs in the bracket) The bracket $-(y - 2)$ means $-1 \times (y - 2)$, which gives $-y + 2$. Then $4y - y = 3y$ and $12 + 2 = 14$.
d. $x^2 + 3x + 2x + 2 = x^2 + 5x + 2$ Only the x terms $3x$ and $2x$ can be collected; x^2 and the number 2 stay as they are.

Answer to Question 3

a. $x^2 + 5x + 3x + 15 = x^2 + 8x + 15$ The four products are x^2, $5x$, $3x$ and 15; the two middle terms add to $8x$.
b. $x^2 - 2x + 4x - 8 = x^2 + 2x - 8$ The middle terms are $-2x$ and $+4x$, which add to $+2x$; the last product $4 \times (-2)$ is -8.
c. $x^2 - 6x - 3x + 18 = x^2 - 9x + 18$ Both numbers are negative, so the middle terms add to $-9x$ and the constant $(-3) \times (-6) = +18$ is positive.
d. $2x^2 + 8x + x + 4 = 2x^2 + 9x + 4$ The first product is $2x \times x = 2x^2$; the middle terms $8x$ and x give $9x$, and the constant is $1 \times 4 = 4$.

Answer to Question 4

a. $x^2 + 4x + 4x + 16 = x^2 + 8x + 16$ Write the bracket twice: the middle terms $4x + 4x$ give $8x$ and the constant is $4^2 = 16$.
b. $x^2 - 5x - 5x + 25 = x^2 - 10x + 25$ The middle terms are both $-5x$, giving $-10x$, and $(-5)^2 = +25$.
c. $x^2 - 6x + 6x - 36 = x^2 - 36$ (the middle terms cancel) The brackets differ only in their sign, so $-6x$ and $+6x$ cancel and the answer is $x^2 - 6^2$.
d. $9x^2 + 6x + 6x + 4 = 9x^2 + 12x + 4$ Here $(3x)^2 = 9x^2$, the middle terms are $2 \times 3x \times 2 = 12x$ and the constant is $2^2 = 4$.

Answer to Question 5

a. HCF is 3: $3(2x + 3)$ Dividing each term by 3 leaves $2x$ and 3. Check: $3 \times 2x + 3 \times 3 = 6x + 9$.
b. HCF is 4: $4(2y - 3)$ Dividing by 4 leaves $2y$ and -3. Check: $4 \times 2y - 4 \times 3 = 8y - 12$.
c. HCF is x: $x(x + 5)$ There is no common number other than 1, but x divides both terms, leaving x and 5 inside the bracket.
d. HCF is $6a$: $6a(2b + 3)$ The HCF of 12 and 18 is 6, and a appears in both terms but b does not, so the HCF is $6a$. Check: $6a \times 2b + 6a \times 3 = 12ab + 18a$.

Answer to Question 6

a. No. $5(x + 2) = 5x + 10$, so the claim is **wrong**. The 5 must multiply both terms in the bracket, not only the x, so the correct expansion ends in $+10$.
b. $2x(x + 3)$. Check: $2x \times x + 2x \times 3 = 2x^2 + 6x$. Correct. The HCF of $2x^2$ and $6x$ is $2x$, because 2 divides both numbers and the lowest power of x is x.
c. $x^2 - 49 = x^2 - 7^2 = (x + 7)(x - 7)$ Since $49 = 7^2$, this is a difference of two squares $a^2 - b^2$ with $a = x$ and $b = 7$.
d. $4x^2 - 9 = (2x)^2 - 3^2 = (2x + 3)(2x - 3)$ Since $4x^2 = (2x)^2$ and $9 = 3^2$, use $a^2 - b^2 = (a + b)(a - b)$ with $a = 2x$ and $b = 3$. Check: $4x^2 - 6x + 6x - 9 = 4x^2 - 9$.

Answer to Question 7

a. $(x + 6)(x + 2) = x^2 + 2x + 6x + 12 = x^2 + 8x + 12$ Multiply each term of the first bracket by each term of the second, then collect $2x + 6x = 8x$. This shows the area is $x^2 + 8x + 12$ m².
b. $16 + 32 + 12 = 60$. Check: $10 \times 6 =$ **60 m²**. When $x = 4$ the sides are 10 m and 6 m. The expression and the actual lengths give the same value, which confirms the expansion: the area is 60 m².
c. $2(x + 6) + 2(x + 2) = 2x + 12 + 2x + 4 = 4x + 16$ A rectangle has two lengths

and two widths, so add twice each side and collect the like terms: the perimeter is $(4x + 16)$ m.

d. When $x = 4$ the perimeter is $16 + 16 = 32$ m. Cost $= 32 \times 8.50 =$ **£272** First find the perimeter, then multiply by the price per metre. The fencing costs £272.

Answer to Question 8

a. $(2x + 1)(x + 4) = 2x^2 + 8x + x + 4 = 2x^2 + 9x + 4$ Multiply out the four products $2x^2$, $8x$, x and 4, then collect $8x + x = 9x$. This proves the result.

b. Expression: $50 + 45 + 4 = 99$. Lengths: $11 \times 9 =$ **99 m²**. They agree. When $x = 5$ the sides are $2 \times 5 + 1 = 11$ m and $5 + 4 = 9$ m, so both methods give an area of 99 m².

c. $99 \div 20 = 4.95$, so **5 tins**. Four tins only cover 80 m², which is not enough, so we must round up: the caretaker needs 5 tins.

d. $5 \times 14.20 =$ **£71** Multiply the number of tins by the price of one tin: the total cost is £71.

Answer to Question 9

a. Side $= x + 3 + 3 = x + 6$ There is a 3 cm border on the left and on the right of the picture, so 6 cm is added to the side x.

b. $(x + 6)^2 = x^2 + 6x + 6x + 36 = x^2 + 12x + 36$ The frame is a square, so its area is $(x + 6)(x + 6)$; the two middle terms $6x$ add to $12x$ and $6^2 = 36$.

c. Border area $= (x^2 + 12x + 36) - x^2 = 12x + 36$ The border is the whole frame without the picture. The picture has area x^2, which cancels, leaving $12x + 36$ cm².

d. $12 \times 20 + 36 = 276$. Check: $26^2 - 20^2 = 676 - 400 =$ **276 cm²**. The frame has side 26 cm and the picture has side 20 cm, so both methods agree: the border is 276 cm².

Answer to Question 10

a. $2(x + 2) + 3x = 2x + 4 + 3x = 5x + 4$ Each adult ticket costs $x + 2$ pounds, so two of them cost $2(x + 2)$; add $3x$ for the children's tickets. The total is $£(5x + 4)$.

b. $5 \times 3 + 4 =$ **£19** Substitute $x = 3$: $15 + 4 = 19$. The Brown family pays £19.

c. $4(x + 2) + 6x = 4x + 8 + 6x = 10x + 8 = 2(5x + 4)$ The HCF of 10 and 8 is 2, so $10x + 8 = 2(5x + 4)$. The Greens pay $£2(5x + 4)$.

d. The Green family buys exactly twice as many tickets, so they pay exactly twice as much as the Browns. The bracket $5x + 4$ is exactly the Browns' total, and it is multiplied by 2, so the Greens pay double for any ticket price.

Answer to Question 11

a. $(100+3)(100-3) = 100^2 - 3^2 = 10000 - 9 =$ **9 991** Here $a = 100$ and $b = 3$, so $a^2 - b^2$ is easy to work out mentally. The answer is 9 991.
b. $50^2 + 2 \times 50 \times 2 + 2^2 = 2500 + 200 + 4 =$ **2 704** Use $(a+b)^2 = a^2 + 2ab + b^2$ with $a = 50$ and $b = 2$. So $52^2 = 2704$.
c. $(50-2)(50+2) = 2500 - 4 =$ **2 496** Write the product as $(50-2)(50+2)$, a difference of two squares, so it equals $50^2 - 2^2 = 2496$.
d. $(100-1)^2 = 10000 - 200 + 1 =$ **9 801** Use $(a-b)^2 = a^2 - 2ab + b^2$ with $a = 100$ and $b = 1$. So $99^2 = 9801$.

Answer to Question 12

a. $x + 3$ Each box contains x muffins and 3 cookies, so one box holds $x + 3$ items.
b. $4(x+3) + 6(x+3) = 4x + 12 + 6x + 18 = 10x + 30$ There are $4 + 6 = 10$ boxes, each with $x + 3$ items. Expanding and collecting gives $10x + 30$ items.
c. $10x + 30 = 10(x+3)$, which is 10 boxes of $x + 3$ items. The HCF of 10 and 30 is 10, so the total factorises as $10(x+3)$: ten boxes of $x + 3$ items.
d. $10 \times (9+3) =$ **120 items** Use the factorised form: $9 + 3 = 12$ items per box and 10 boxes, so the baker packs 120 items in a day.

Answer to Question 13

a. $x^2 + 3x + 2x + 6 = x^2 + 5x + 6$ The four products are x^2, $3x$, $2x$ and 6; collecting the middle terms gives $5x$.
b. $x(x^2 + 5x + 6) = x^3 + 5x^2 + 6x$ Volume = length × width × height, so multiply the answer to part a) by x. Each term gains one power of x.
c. Expression: $64 + 80 + 24 = 168$. Lengths: $4 \times 6 \times 7 =$ **168 cm³**. Both methods give the same value, so the volume is 168 cm³ when $x = 4$.
d. $188 \times 0.2 = 37.6$ p, so about **38p**. Multiply the surface area by the cost per cm². Check the surface area: $2(4 \times 6 + 4 \times 7 + 6 \times 7) = 2 \times 94 = 188$ cm². The card costs about 38p.

Answer to Question 14

a. $4(2x+5) = 8x + 20$ Jade saves the same amount every week, so multiply by 4: her total is £$(8x + 20)$.
b. $3(x+8) = 3x + 24$ Ben saves for 3 weeks, so his total is £$(3x + 24)$.
c. $8x + 20 + 3x + 24 = 11x + 44 = 11(x+4)$ Add the two totals and collect like terms: $11x + 44$. The HCF of 11 and 44 is 11, so the total is £$11(x+4)$.
d. $11 \times (6+4) =$ **£110** Substitute $x = 6$ into the factorised form: $11 \times 10 = 110$. Together they save £110.

Answer to Question 15

a. $(x-2)(x+3) = x^2+3x-2x-6 = x^2+x-6$. $(x+2)(x-3) = x^2-x-6$. $(x-2)(x-3) = x^2-5x+6$. In each case multiply the four pairs of terms and collect the two x terms; take care that a negative times a positive is negative.

b. The coefficient of x is the **sum** of the two numbers and the constant is their **product**. For $(x+a)(x+b)$ the middle terms are $bx+ax = (a+b)x$ and the last term is ab, which explains the pattern.

c. Sum 11, product 28: $x^2+11x+28$ The numbers are 4 and 7: their sum is 11 and their product is 28, so the expansion is $x^2+11x+28$.

d. Sum $= 3$, product $= -40$: $x^2+3x-40$. Check: $x^2+8x-5x-40 = x^2+3x-40$. Correct. The numbers are -5 and 8: $-5+8=3$ and $-5\times 8 = -40$. The prediction and the full expansion agree.

Answer to Question 16

a. Ali wrong, Bea wrong, Cal **right**, Dev wrong. Expand each one yourself and compare. Only Cal has every term correct; each of the others has a missing term or a wrong sign.

b. Ali: $(x+3)(x+3) = x^2+6x+9$. Bea: $3x-12$. Dev: $-3x+6$ (a negative times a negative is positive). Ali forgot the middle term, Bea forgot to multiply the 4 by 3, and Dev forgot that $-3\times(-2) = +6$.

c. When $x=1$: $(1+3)^2 = 16$ but $1+9=10$. The two expressions are not equivalent: the middle term $6x$ was forgotten. Substituting a number shows the two sides have different values, so $(x+3)^2$ and x^2+9 cannot be equivalent.

d. Cal's: $(x-2)(x+2) = x^2-4$. Cal's working has the form $(a-b)(a+b) = a^2-b^2$ with $a=x$ and $b=2$, so it is the difference of two squares.

Answer to Question 17

a. HCF is $3x$: $6x^2+9x = 3x(2x+3)$ The HCF of 6 and 9 is 3 and both terms contain x, so take out $3x$. Check: $3x\times 2x + 3x\times 3 = 6x^2+9x$.

b. Other side $= (2x+3)$ **m** Area = length × width, and one side is $3x$, so the other side is the bracket left after factorising.

c. $2(3x+2x+3) = 10x+6$ metres Add the two different sides and double: $3x+2x+3 = 5x+3$, so the perimeter is $(10x+6)$ m.

d. Sides $3x = 6$ m and $2x+3 = 7$ m. Area $= 42$ m^2. Check: $6\times 4+18 = 24+18 = 42$. Substituting into the expression and multiplying the side lengths both give 42 m², so the patio is 6 m by 7 m.

e. $42\times 7.50 =$ **£315** Multiply the area by the price per square metre: the slabs cost £315.

Answer to Question 18

a. $n = 2$: $(2+4)^2 = 36$, $36 - 16 = 20$, $20 \div 2 = 10$. $n = 5$: $81 - 16 = 65$, $65 \div 5 = 13$. $n = 10$ also works: $14^2 = 196$, $196 - 16 = 180$ and $180 \div 10 = 18$. All the entries in the table are correct.

b. $\dfrac{(n+4)^2 - 16}{n}$ Follow the steps in order: adding 4 gives $n + 4$, squaring gives $(n+4)^2$, subtract 16, then divide the whole result by n.

c. $(n+4)^2 - 16 = n^2 + 8n + 16 - 16 = n^2 + 8n = n(n+8)$ The numbers 16 and -16 cancel, and n is a common factor of n^2 and $8n$, giving $n(n+8)$.

d. $\dfrac{n(n+8)}{n} = n + 8$ for every $n \neq 0$. The factor n on the top cancels with the n on the bottom, so the final answer is always 8 more than the starting number.

e. Dividing by zero is impossible, so $n = 0$ is not allowed. When $n = 0$ the last step asks us to divide 0 by 0, which has no value, so the trick only works for non-zero numbers.

Answer to Question 19

a. Width $x + 4$, height $x + 8$ (2 cm is added on each side). A 2 cm border on both sides adds $2 + 2 = 4$ cm to the width and 4 cm to the height, so the poster is $(x+4)$ cm by $(x+8)$ cm.

b. $(x+4)(x+8) = x^2 + 8x + 4x + 32 = x^2 + 12x + 32$ Multiply the four products x^2, $8x$, $4x$ and 32, then collect $8x + 4x = 12x$.

c. Picture area $= x(x+4) = x^2 + 4x$. Border $= x^2 + 12x + 32 - x^2 - 4x = 8x + 32 = 8(x+4)$ Subtract the picture area from the poster area; the x^2 terms cancel and $12x - 4x = 8x$. The HCF of 8 and 32 is 8.

d. $8 \times 16 = 128$. Check: $16 \times 20 - 12 \times 16 = 320 - 192 =$ **128 cm²**. When $x = 12$ the poster is 16 cm by 20 cm and the picture is 12 cm by 16 cm, so the border area is 128 cm² both ways.

e. $128 \times 0.5 =$ **64p** Multiply the border area by the cost per cm²: the paper costs 64p.

Answer to Question 20

a. $(2k+3)^2 = 4k^2 + 12k + 9$ and $(2k+1)^2 = 4k^2 + 4k + 1$. Use $(a+b)^2 = a^2 + 2ab + b^2$: for $2k + 3$ the middle term is $2 \times 2k \times 3 = 12k$, and for $2k + 1$ it is $4k$.

b. Subtract: $(4k^2 + 12k + 9) - (4k^2 + 4k + 1) = 8k + 8$. The $4k^2$ terms cancel, $12k - 4k = 8k$ and $9 - 1 = 8$, which proves the result.

c. $8k + 8 = 8(k+1)$, which is a multiple of 8. Since 8 is a factor of $8(k+1)$ and $k + 1$ is a whole number, the difference is always a multiple of 8.

d. $49 - 25 = 24 = 8 \times 3$. (Here $k = 2$, so $8(k+1) = 24$.) The odd numbers 5 and 7

give $2k+1=5$, so $k=2$, and the formula predicts $8\times 3=24$, which matches.

e. $(51-49)(51+49)=2\times 100=200=8\times 25$. Using $a^2-b^2=(a-b)(a+b)$ avoids squaring large numbers. Since $200=8\times 25$, it is a multiple of 8.