Answers - Direct and inverse proportion

Year: Year 9

Answer to Question 1

a. $k = y \div x = 12 \div 4 =$ **3** In direct proportion $y \div x$ is always the same, so divide the given value of y by the value of x.
b. $y = 3x$ Replace k by 3 in $y = kx$. This equation works for every pair of values.
c. $y = 3 \times 9 =$ **27** Substitute $x = 9$ into the equation: when $x = 9$, $y = 27$.
d. $45 = 3x$, so $x = 45 \div 3 =$ **15** Substitute $y = 45$ and solve by dividing both sides by 3. Check: $3 \times 15 = 45$.

Answer to Question 2

a. $k = xy = 8 \times 6 =$ **48** In inverse proportion the product $x \times y$ is always the same, so multiply the given pair.
b. $y = \frac{48}{x}$ Replace k by 48 in $y = \frac{k}{x}$.
c. $y = 48 \div 12 =$ **4** Substitute $x = 12$: when $x = 12$, $y = 4$. Check: $12 \times 4 = 48$.
d. $16 = \frac{48}{x}$, so $x = 48 \div 16 =$ **3** Multiply both sides by x to get $16x = 48$, then divide by 16. Check: $3 \times 16 = 48$.

Answer to Question 3

a. 1 pen $= 3.45 \div 5 = £0.69$. 12 pens $= 12 \times 0.69 =$ **£8.28** Divide by 5 to find the price of one pen, then multiply by 12. Twelve pens cost £8.28.
b. 1 kg $= 11.76 \div 8 = £1.47$. 5 kg $= 5 \times 1.47 =$ **£7.35** Find the price of 1 kg by dividing by 8, then multiply by 5. Five kilograms cost £7.35.
c. 9 bottles in 6 s is 1.5 per second. 120 s $\times 1.5 =$ **180 bottles** Change 2 minutes into 120 seconds so the units match, then use the rate per second. The machine fills 180 bottles.
d. 1 notebook $= 9.45 \div 7 = £1.35$. $16.20 \div 1.35 =$ **12 notebooks** Find the price of one notebook, then see how many times it goes into £16.20. Exactly 12 notebooks can be bought.

Answer to Question 4

a. **Direct**: doubling the tickets doubles the cost. The cost is always 7.5 times the number of tickets, so the ratio is constant: direct proportion.
b. **Inverse**: doubling the speed halves the time ($speed \times time = 120$). Speed × time

is always 120 km, so the product is constant: inverse proportion.

c. **Inverse**: twice as many workers take half as long. The total amount of digging stays the same, so workers × days is constant: inverse proportion.

d. **Neither**: height does not grow in a constant ratio with age. A baby is not half the height of a two-year-old, and growth slows down, so there is no constant ratio: neither.

Answer to Question 5

a. $k = 6 \div 2 = 3$. When $x = 5$, $y = 15$. When $y = 21$, $x = 7$. When $x = 10$, $y = 30$. Missing: **15, 7, 30**. For direct proportion, $y = 3x$: multiply x by 3 to find y, and divide y by 3 to find x.

b. $k = 2 \times 20 = 40$. When $x = 4$, $y = 10$. When $y = 5$, $x = 8$. When $x = 10$, $y = 4$. Missing: **10, 8, 4**. For inverse proportion, $xy = 40$: divide 40 by the known value to find the missing one.

Answer to Question 6

a. Direct. $y = 5 \times 0.6 =$ **3** The equation has the form $y = kx$ with $k = 5$, so it shows direct proportion.

b. Inverse. $9 = 36 \div x$, so $x = 36 \div 9 =$ **4** The equation has the form $y = \frac{k}{x}$ with $k = 36$, so it shows inverse proportion.

c. No. When $x = 1$, $y = 7$; when $x = 2$, $y = 12$. Doubling x does not double y (12 is not 14), so it is **neither**. The graph of $y = 5x + 2$ is a straight line that does not pass through the origin, which is why it is not direct proportion.

d. Direct, with $k = \frac{1}{4} =$ **0.25** Dividing by 4 is the same as multiplying by $\frac{1}{4}$, so the equation is $y = 0.25x$.

Answer to Question 7

a. Scale factor $30 \div 12 = 2.5$: flour $= 450$ g, butter $= 225$ g, sugar $= 300$ g. All three ingredients are multiplied by the same scale factor of 2.5, because 30 cookies is 2.5 times 12 cookies.

b. Scale factor $18 \div 12 = 1.5$: flour $= 270$ g, butter $= 135$ g, sugar $= 180$ g. Use the scale factor 1.5, because 18 is one and a half times 12.

c. Sugar per cookie $= 120 \div 12 = 10$ g. $500 \div 10 =$ **50 cookies**. Sugar is the limiting ingredient. Each cookie needs 10 g, so 500 g makes 50 cookies.

d. 450 g $= 0.45$ kg. $0.45 \times 1.10 =$ **£0.495**, so about **50p**. Change grams to kilograms by dividing by 1000, then multiply by the price per kilogram. The flour costs about 50p.

Answer to Question 8

a. $6.4 \div 100 =$ **0.064 litres per km** Divide the petrol by the distance to find the amount used for 1 km.
b. $575 \times 0.064 =$ **36.8 litres** Multiply the petrol per kilometre by the distance of 575 km. The trip uses 36.8 litres.
c. $36.8 \times 1.45 =$ **£53.36** Multiply the litres by the price per litre. The petrol for the trip costs £53.36.
d. $40 \div 0.064 =$ **625 km** Divide the size of the tank by the petrol used per kilometre. A full tank lasts 625 km.

Answer to Question 9

a. $3 \times 8 =$ **24 painter-days** Multiply the number of painters by the number of days to find the total amount of work.
b. $24 \div 1 = 24$; $24 \div 2 = 12$; $24 \div 4 = 6$; $24 \div 6 = 4$; and $24 \div 2 = 12$ painters for 2 days. Missing: **24, 12, 6, 4, 12**. For each column divide 24 painter-days by the number of painters, or by the number of days.
c. $24 \div 5 = 4.8$, so **5 painters** (the number must be a whole number, rounding up). Four painters would take 6 days, which is too long, so we must round up to 5 painters (they take 4.8 days).
d. Painters get in each other's way, or some work faster than others, so the model of equal rates may not hold. The model assumes that all painters work at the same rate all the time; in real life weather and breaks also affect the time.

Answer to Question 10

a. $250 \times 1.15 =$ **€287.50** Pounds and euros are in direct proportion, so multiply by the rate. £250 buys €287.50.
b. $172.50 \div 1.15 =$ **£150** To change euros into pounds, divide by the rate. The souvenirs are worth £150.
c. She changes $250 - 3 = £247$, so $247 \times 1.15 =$ **€284.05** Take off the charge first, because it is paid in pounds, then convert what is left. Sam receives €284.05.
d. $3.45 \div 1.15 =$ **£3.00** Divide the euro price by the rate. The hot chocolate costs £3.00.

Answer to Question 11

a. $60 \times 4 = 80 \times 3 = 48 \times 5 = 240$: the product is always the same constant 240. In inverse proportion speed × time is constant. Here every pair gives 240, which is the length of the journey.
b. $240 \div 2.5 =$ **96 km/h** Rearrange speed × time = 240 to find the speed: $240 \div 2.5 = 96$ km/h.

c. $t = \dfrac{240}{s}$ Since $s \times t = 240$, dividing both sides by s gives the formula.
d. $240 \div 64 = 3.75$ h $= 3$ h 45 min. $07{:}15 + 3{:}45 =$ **11:00**. The decimal 0.75 h is three quarters of an hour, which is 45 minutes. The coach arrives at 11:00.

Answer to Question 12

a. $30 \div 10 = 3$, so $3 \times 3 =$ **9 pizzas** Thirty children is three times ten children, so three times as many pizzas are needed: 9 pizzas.
b. $24 \times 0.3 = 7.2$, so **8 pizzas** Each child needs $3 \div 10 = 0.3$ of a pizza. Since 7 pizzas would not be enough, round up to 8 pizzas.
c. $8 \times 8.50 =$ **£68** Multiply the number of pizzas by the price of one pizza: the pizzas cost £68.
d. $50 \div 8.50 = 5.88$, so 5 pizzas, which feed $5 \div 0.3 = 16.7$, so **16 children** (whole children). Six pizzas would cost £51, which is too much, so the class can buy 5 pizzas and feed 16 children.

Answer to Question 13

a. $4 \times 30 =$ **120 hamster-days** Four hamsters eating for 30 days is the same amount of food as one hamster eating for 120 days.
b. $120 \div 6 =$ **20 days** Share the 120 hamster-days between 6 hamsters: the bag lasts 20 days.
c. $120 \div 5 =$ **24 days** Share the 120 hamster-days between 5 hamsters: the bag lasts 24 days.
d. $6.60 \div 120 =$ **£0.055**, which is 5.5p. Divide the cost of the bag by the 120 hamster-days of food. One hamster costs 5.5p a day to feed.

Answer to Question 14

a. $1800 \div 15 = 120$ min $=$ **2 hours** Divide the volume of the tank by the rate of one pump to find the time in minutes, then divide by 60 for hours.
b. $120 \div 2 =$ **60 minutes** Two pumps fill twice as fast, so the time is halved: 60 minutes.
c. $120 \div 30 =$ **4 pumps** One pump would take 120 minutes; to finish in 30 minutes we need $120 \div 30$ pumps working together.
d. **Inversely**: more pumps means less time, and (pumps $\times$ time) is constant at 120. Doubling the number of pumps halves the time, which is the sign of inverse proportion.

Answer to Question 15

a. $3 \div 2 = 6 \div 4 = 9 \div 6 = 12 \div 8 = 1.5$, so $C = 1.5m$. All the ratios cost ÷ mass are equal to 1.5, so the cost is directly proportional to the mass with $k = 1.5$.
b. $3.50 \div 2 = 1.75$ but $6.00 \div 4 = 1.50$. The ratios differ, so it is not proportional. In direct proportion the ratio must stay the same, but Shop B's ratio changes, so it is not proportional.
c. The cost rises by £2.50 for every 2 kg, so £1.25 per kg: $C = 1.25m + 1$. For 10 kg: $12.50 + 1 =$ **£13.50**. The £1 is the fixed delivery charge; the price for 10 kg at Shop B is £13.50.
d. At 10 kg: A $= £15.00$, B $= £13.50$, so **Shop B** is cheaper. At 2 kg: A $= £3.00$, B $= £3.50$, so **Shop A** is cheaper. The fixed charge makes Shop B dearer for small orders, but its lower price per kilogram makes it cheaper for large orders.
e. $1.5m = 1.25m + 1$, so $0.25m = 1$ and $m =$ **4 kg**. Both shops cost £6.00 for 4 kg. Set the two formulas equal and solve. Both shops charge £6.00 for 4 kg.

Answer to Question 16

a. $6 \times 10 =$ **60 bricklayer-days** Multiply the number of bricklayers by the days to find the total work needed.
b. $60 \div 4 =$ **15 days** Share the 60 bricklayer-days between 4 workers: they take 15 days.
c. $60 \div 4 =$ **15 bricklayers** Divide the 60 bricklayer-days by the 4 days available: 15 bricklayers are needed.
d. After 3 days, $6 \times 3 = 18$ bricklayer-days are done and $60 - 18 = 42$ remain. $42 \div 4 =$ **10.5 days**. Work out how much has been done, subtract from the total, then share the rest between the 4 who are left.
e. $3 + 10.5 =$ **13.5 days** in total. Add the 3 days already worked to the 10.5 extra days: the wall takes 13.5 days altogether.

Answer to Question 17

a. $20 \times 45 = 30 \times 30 = 40 \times 22.5 = 60 \times 15 = 900$, so $k =$ **900**. Every product of speed and time is 900, so time is inversely proportional to speed and $t = \frac{900}{s}$.
b. $900 \div 20 =$ **45 km/h** Substitute $t = 20$ and solve $20 = \frac{900}{s}$, giving $s = 45$.
c. $900 \div 25 = 36$ min, so it arrives at **08:26**. Find the journey time from the formula, then add it to 07:50. The bus arrives at 08:26.
d. $900 \div 12 = 75$ min, which is **more** than an hour, so she will not. At 12 km/h the journey takes 75 minutes, which is 1 hour 15 minutes, so Layla will be late if she has only an hour.
e. Traffic, stops and speed limits change the average speed during the journey. The formula uses a constant average speed, but a real journey has traffic lights, junctions and queues.

Answer to Question 18

a. $240 \div 8 =$ **30 per minute** Divide the flyers by the time to find the rate of one printer: 30 flyers per minute.
b. $1500 \div 30 =$ **50 minutes** Divide the order by the rate: one printer takes 50 minutes.
c. $50 \div 3 = 16.67$ min = **16 minutes 40 seconds** Three printers share the work, so divide 50 minutes by 3. This is 16 minutes and 40 seconds.
d. $50 \div 10 =$ **5 printers** The work is 50 printer-minutes; finishing in 10 minutes needs 5 printers.
e. $1500 \times 0.04 = 60$, plus 5 = **£65** Work out the cost of the flyers, then add the fixed set-up charge. The order costs £65.

Answer to Question 19

a. When $x = 3$: $y = 2 \times 3 =$ **6** and $y = 18 \div 3 =$ **6**. Substitute $x = 3$ into each rule. Both rules give the same value at this point.
b. $y = 2x$ is **direct** (a straight line through the origin). $y = 18/x$ is **inverse** (a curve that falls as x increases and never touches the axes). A straight line through the origin shows direct proportion; a curve where xy is constant shows inverse proportion.
c. They cross where both values are equal: $x =$ **3**, $y =$ **6**. The two graphs cross where they have the same y-value for the same x-value: at the point $(3, 6)$.
d. $2x = \dfrac{18}{x}$, so $2x^2 = 18$, $x^2 = 9$ and $x = 3$ (taking the positive value). Then $y = 6$. Multiply both sides of the equation by x, divide by 2 and take the square root. Substituting $x = 3$ into $y = 2x$ gives $y = 6$.

Answer to Question 20

a. $18 = k \times 9$, so $k =$ **2**. Substitute the given pair into $y = kx^2$; remember to square x first.
b. $y = 2 \times 25 =$ **50** Square 5 first, then multiply by $k = 2$: $y = 50$.
c. $72 = 2x^2$, so $x^2 = 36$ and $x =$ **6**. Divide both sides by 2 and take the positive square root of 36.
d. $y = 2(2x)^2 = 8x^2$, which is **4 times** as big. Since $(2x)^2 = 4x^2$, doubling x multiplies y by $2^2 = 4$.
e. $w = \dfrac{k}{x^2}$, so $4 = \dfrac{k}{9}$ and $k = 36$. When $x = 6$, $w = 36 \div 36 =$ **1**. Find the new constant from the pair $w = 4$, $x = 3$. When x doubles, w is divided by $2^2 = 4$, which agrees with $w = 1$.