Test answers - Angles in polygons

Year: Year 9 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. Use the rule sum $= (n-2) \times 180^\circ$ with $n = 6$: $(6-2) \times 180 = 720^\circ$ (1 mark). Drawing diagonals from one vertex splits a hexagon into 4 triangles, so its angle sum is $4 \times 180^\circ$.
b. Find the exterior angle first by sharing the full turn between 8 equal angles: $360 \div 8 = 45^\circ$ (1 mark). The interior angle sits on a straight line with it: $180 - 45 = 135^\circ$ (1 mark).
c. The exterior angles of any polygon add up to 360°, so the number of sides is $360 \div 24 =$ **15** (1 mark). The 15 equal exterior angles fill the full turn.

Check (b): $8 \times 135 = 1080 = (8-2) \times 180$. Each interior angle of the 15-sided polygon is $180 - 24 = 156^\circ$.

Answer to Question 2 (4 marks)

a. A pentagon splits into $5 - 2 = 3$ triangles, so its interior angles add up to $3 \times 180 = 540^\circ$. (1 mark)
b. The five angles must add up to 540°, and the two unknown angles are both x: $120 + 100 + 110 + x + x = 540$, which simplifies to $330 + 2x = 540$. (1 mark)
c. Subtract 330 from both sides: $2x = 210$ (1 mark). Divide both sides by 2: $x = 105^\circ$ (1 mark).

Check: $120 + 100 + 110 + 105 + 105 = 540$. Every angle is less than 180°, so the pentagon is convex, which is sensible. A common error is writing x only once in the equation, which would give $x = 210^\circ$, an impossible reflex angle here.

Answer to Question 3 (4 marks)

a. A hexagon has angle sum 720°, so each interior angle of a regular hexagon is $720 \div 6 = 120^\circ$ (1 mark). Three hexagons meeting at a point give $3 \times 120 = 360^\circ$, exactly a full turn, so they fit together with no gap and no overlap (1 mark for clearly stating the total is 360°).
b. The exterior angle of a regular decagon is $360 \div 10 = 36^\circ$, so the interior angle is $180 - 36 = 144^\circ$ (1 mark). Then $360 \div 144 = 2.5$, not a whole number, so a whole number of decagons cannot meet at a point (1 mark).

Two decagons leave a gap of $360 - 288 = 72^\circ$, which is too small for a third decagon. This is why honeycombs use hexagons: only triangles, squares and hexagons tile on their own.

Answer to Question 4 (4 marks)

a. Each turn of 36° is an exterior angle of the path, and the turns must add up to one full turn of 360° for the robot to face its starting direction again. It repeats the command $360 \div 36 =$**10** times. (1 mark)
b. At each corner the interior and exterior angles lie on a straight line: $180 - 36 = 144^\circ$. (1 mark)
c. The path is a regular decagon with 10 sides of 1.5 m, so the robot travels $10 \times 1.5 =$**15 m**. (1 mark)
d. With 45° turns: $360 \div 45 = 8$ equal sides, so it traces a regular **octagon**. (1 mark)

The octagon path would be $8 \times 1.5 = 12$ m long.

Answer to Question 5 (4 marks)

a. At each vertex the interior and exterior angles make a straight line, so the exterior angle is $180 - 130 = 50^\circ$. (1 mark)
b. For a regular polygon the number of sides is $360 \div$ exterior angle $= 360 \div 50 = 7.2$ (1 mark). A polygon must have a whole number of sides, so no regular polygon has an interior angle of 130° and Jake cannot be correct (1 mark).
c. Exterior angle $= 180 - 135 = 45^\circ$, so the number of sides is $360 \div 45 =$**8**; it is a regular octagon. (1 mark)

The regular heptagon (7 sides) has interior angles of about 128.6° and the octagon 135°, so 130° falls between them. Only exterior angles that divide exactly into 360 give real regular polygons.