Worksheet - Angles in polygons

Year: Year 9 · 20

Show your working.

Question 1

Question 1 – Angle sums *(Section A · Skills practice)*

The sum of the interior angles of an n-sided polygon is $(n-2)\times 180^\circ$. Use this rule to complete the following, showing each step.

a. Find the angle sum of a pentagon ($n=5$).
b. Find the angle sum of a heptagon ($n=7$).
c. Find the angle sum of a 12-sided polygon.
d. A polygon has an angle sum of 1620°. How many sides does it have?

Question 2

Question 2 – Missing interior angles *(Section A · Skills practice)*

Work out the size of the angle marked x in each polygon. Write the angle sum first.

a. A quadrilateral has angles 85°, 102°, 96° and x.
b. A pentagon has angles 110°, 95°, 120°, 105° and x.
c. A hexagon has angles 130°, 125°, 118°, 140°, 112° and x.
d. A pentagon has angles 90°, 90°, 90°, x and x.

Question 3

Question 3 – Regular polygons *(Section A · Skills practice)*

A regular polygon has equal sides and equal angles. Find the size of one interior angle of each regular polygon.

a. Regular hexagon
b. Regular octagon
c. Regular decagon (10 sides)
d. Regular 15-sided polygon, giving your answer as a whole number

Question 4

Question 4 – Exterior angles *(Section A · Skills practice)*

The exterior angles of any polygon add up to 360°, and at each vertex interior angle + exterior angle $=180^\circ$. For a regular polygon each exterior angle is $360\div n$.

a. Find one exterior angle of a regular pentagon.

b. Find one exterior angle of a regular 9-sided polygon.
c. A regular polygon has an exterior angle of 30°. How many sides has it?
d. Find the interior angle of the polygon in part (c).

Question 5

Question 5 – How many sides? *(Section A · Skills practice)*

Find the number of sides of each polygon. Show how you decide.

a. A polygon has an angle sum of 1440°.
b. A regular polygon has interior angles of 140°.
c. A regular polygon has exterior angles of 15°.
d. A polygon has an angle sum of 2340°.

Question 6

Question 6 – Angles and algebra *(Section A · Skills practice)*

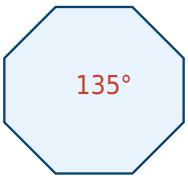
Form and solve an equation in each part.

a. The angles of a quadrilateral are x, $x + 20$, $2x$ and $2x + 40$ degrees. Find x and write down all four angles.
b. The five equal interior angles of a regular pentagon are each $(3x + 6)^\circ$. Find x.
c. Four angles of a hexagon are each 100°. The other two are equal. Find them.

Question 7

Question 7 – Tiling the kitchen floor *(Section B · Real-life practice)*

Maya is tiling a kitchen floor. The shop sells regular tiles in these shapes: equilateral triangle, square, regular pentagon, regular hexagon and regular octagon. For tiles to fit together with no gaps, the angles that meet at each point must add up to 360°.



a. Find the interior angle of each of the five tile shapes.
b. Show that three hexagonal tiles meeting at a point leave no gap.
c. Show that two octagons and one square also fit together at a point.
d. Explain why regular pentagons alone will not tile the floor.
e. Maya covers a 3.6 m by 2.4 m floor with square tiles of side 30 cm. How many tiles does she need?

Question 8

Question 8 – The stop sign *(Section B · Real-life practice)*

A stop sign is a regular octagon with each side 30 cm long. The council is ordering signs for a new housing estate.

a. Find the perimeter of one sign.
b. Find the size of each interior angle and each exterior angle.
c. A workman walks once around the edge of the sign, turning at each corner. Through what total angle does he turn?
d. Each sign costs £42.50 and the council orders 14. Find the total cost.

Question 9

Question 9 – Inside the beehive *(Section B · Real-life practice)*

The cells of a honeycomb are regular hexagons. A beekeeper measures one cell and finds that each side is 6 mm long.

a. Find the perimeter of one cell.
b. Find the interior angle of a hexagon.
c. Explain why three cells meet at each corner without any gaps.
d. Each frame holds 3 200 cells. A hive has 9 frames. How many cells are in the hive?
e. A beekeeper says that the cells could not be regular pentagons. Is she right? Explain.

Question 10

Question 10 – The football pattern *(Section B · Real-life practice)*

A classic football is covered with 12 black regular pentagons and 20 white regular hexagons. At each corner one pentagon meets two hexagons.

a. Write down the interior angle of a regular pentagon and of a regular hexagon.
b. Add the three angles that meet at a corner.
c. Why is the answer not 360°? What does this tell you about the surface?
d. Each pentagon has 5 sides and each hexagon has 6. Count the sides on all the panels together, then use the fact that each edge is shared by two panels to find the number of edges.

Question 11

Question 11 – The pentagon flowerbed *(Section B · Real-life practice)*

A school garden has a flowerbed in the shape of a regular pentagon $ABCDE$ with sides of 2 m. A fence will go round the edge and costs £12 per metre.

a. Find the cost of the fence.
b. Write down the interior angle at A.
c. The gardener lays a path along the diagonal AC. Triangle ABC is isosceles with $AB = BC$. Find angle BAC.
d. The interior angle BAE is 108°. Find angle CAE, between the path AC and the side AE.

Question 12

Question 12 – The turtle robot *(Section B · Real-life practice)*

A robot is programmed to move forward 2 m, then turn left through 40°, and to repeat this command until it returns to its start.

a. How many times does the robot repeat the command?
b. What regular polygon does the path make?
c. Find the total distance the robot travels.
d. The program is changed so that the robot turns through 72° instead. What shape is made and what is its interior angle?

Question 13

Question 13 – Fencing the field *(Section B · Real-life practice)*

A farmer's field has five corners. A surveyor measures four of the interior angles as 98°, 121°, 104° and 117°. The five sides measure 40 m, 55 m, 62 m, 48 m and 51 m.

a. Work out the fifth interior angle.
b. Find the perimeter of the field.
c. Fencing costs £7.50 per metre. Find the cost of fencing the whole field.
d. The farmer leaves a 4 m gap for a gate. Find the new cost.

Question 14

Question 14 – The round window *(Section B · Real-life practice)*

A church has a window in the shape of a regular 12-sided polygon. Twelve identical triangular panes meet at the centre.

a. Find each exterior angle of the window.
b. Find each interior angle of the window.
c. Each pane is an isosceles triangle with the apex at the centre. Find the apex angle.
d. Find the two equal base angles of each pane and check that they add up to the interior angle of the window.

Question 15

Question 15 – The polygon fact file *(Section C · Problem solving)*

A teacher gives the class a fact file about regular polygons. Some entries are missing.

Polygon	Sides	Angle sum	Interior angle	Exterior angle
Pentagon	5	?	108°	?
Hexagon	6	?	?	60°
Octagon	8	1080°	?	?
?	10	1440°	144°	?
?	?	?	?	24°

a. Complete the table.
b. Which regular polygon has an interior angle exactly twice its exterior angle? Show your working.
c. Which regular polygon has an interior angle that is 5 times its exterior angle?
d. Explain why no regular polygon has an interior angle of 170° and an exterior angle of 20° at the same time as having fewer than 15 sides.

Question 16

Question 16 – Inside a regular pentagon *(Section C · Problem solving)*

$ABCDE$ is a regular pentagon. The diagonals AC and AD are drawn from A.

a. Write down angle ABC.
b. Triangle ABC is isosceles. Find angle BAC and angle BCA.
c. Find angle CAD using the fact that angle $BAE = 108^\circ$ and that triangle ADE is congruent to triangle ABC.
d. Show that the three angles at A in triangles ABC, ACD and ADE add up to 108°.
e. Triangle ACD is isosceles with $AC = AD$. Find angle ACD.

Question 17

Question 17 – Spot the mistakes *(Section C · Problem solving)*

Four pupils make claims about polygons. Decide which are true. If a claim is false, explain why and give the correct fact.

Pupil	Claim
Ali	The interior angles of a heptagon add up to 900°.
Bea	A regular polygon can have an interior angle of 100°.
Cal	Each exterior angle of a regular 20-sided polygon is 20°.
Dev	A polygon can have an angle sum of 1000°.

a. Which pupils are correct?
b. Correct the wrong claims.
c. What is the largest whole-number exterior angle that a regular polygon can have, and which polygon is it?

Question 18

Question 18 – A mitre joint for a gazebo *(Section C · Problem solving)*

A carpenter is building a garden gazebo with a regular octagonal frame. The rails meet at each corner with a mitre joint: each rail is cut at half the interior angle.

a. Find the interior angle of a regular octagon.
b. What angle does the carpenter cut on each rail?
c. Each side is 1.5 m. Find the perimeter of the frame.
d. Posts are placed every 0.5 m around the perimeter. How many posts are needed?
e. Each post costs £3.50. Find the total cost of the posts.

Question 19

Question 19 – Designing a coin *(Section C · Problem solving)*

A designer is sketching a seven-sided coin. She starts with a regular heptagon with sides of 2.4 cm. Give angles to 1 decimal place where necessary.

a. Find the sum of the interior angles of a heptagon.
b. Find each interior angle and each exterior angle.
c. Find the perimeter of the sketch.
d. Seven of these coins are placed edge to edge around a central point so that one corner of each touches the centre. Explain why this cannot fit with no gaps.
e. How big is the gap?

Question 20

Question 20 – Challenge: a polygon with one right angle *(Section D · Challenge)*

A polygon has n sides. All its interior angles are 150° except one, which is 90°.

a. Write down an expression for the sum of the interior angles in terms of n.
b. Write down a second expression for the sum using the angle sizes.
c. Form and solve an equation to find n.
d. Check your answer by finding the sum both ways.