Answers - Angles in polygons

Year: Year 9

Answer to Question 1

a. $(5-2)\times 180 = 3\times 180 = 540^\circ$ A pentagon splits into 3 triangles from one vertex, so its angles add up to three lots of 180°.
b. $(7-2)\times 180 = 5\times 180 = 900^\circ$ A heptagon has 7 sides and splits into 5 triangles, so the angle sum is 900°.
c. $(12-2)\times 180 = 10\times 180 = 1800^\circ$ Use $n = 12$ in the rule: ten triangles of 180° give 1800°.
d. $1620 \div 180 = 9$, so $n - 2 = 9$ and $n =$ **11 sides** Work backwards: divide the angle sum by 180 to find the number of triangles, then add 2. The polygon has 11 sides.

Answer to Question 2

a. Sum $= 360^\circ$. Known: $85 + 102 + 96 = 283$. $x = 360 - 283 = 77^\circ$ A quadrilateral has 4 sides, so its angles add up to 360°; subtract the three known angles.
b. Sum $= 540^\circ$. Known: $110 + 95 + 120 + 105 = 430$. $x = 540 - 430 = 110^\circ$ A pentagon has an angle sum of 540°; subtracting the four known angles leaves $x = 110^\circ$.
c. Sum $= 720^\circ$. Known: $130 + 125 + 118 + 140 + 112 = 625$. $x = 720 - 625 = 95^\circ$ A hexagon has an angle sum of $4\times 180 = 720^\circ$; the missing angle is 95°.
d. Sum $= 540^\circ$. Known: 270. $2x = 270$, so $x = 135^\circ$ The three right angles make 270°, so the two equal angles share the remaining 270°: each one is 135°.

Answer to Question 3

a. $(6-2)\times 180 \div 6 = 720 \div 6 = 120^\circ$ Find the angle sum first, then share it equally between the 6 equal angles.
b. $(8-2)\times 180 \div 8 = 1080 \div 8 = 135^\circ$ An octagon has an angle sum of 1080°, shared equally between 8 angles, so each is 135°.
c. $(10-2)\times 180 \div 10 = 1440 \div 10 = 144^\circ$ Check with the exterior angle: $360 \div 10 = 36$ and $180 - 36 = 144$, which agrees.
d. $(15-2)\times 180 \div 15 = 2340 \div 15 = 156^\circ$ Check with the exterior angle: $360 \div 15 = 24$ and $180 - 24 = 156$. Each angle is 156°.

Answer to Question 4

a. $360 \div 5 = 72^\circ$ The five equal exterior angles share the full turn of 360°, so each is

72°.

b. $360 \div 9 = 40^\circ$ Nine equal exterior angles share 360°, so each exterior angle is 40°.

c. $360 \div 30 =$ **12 sides** Each turn is 30° and the turns must add up to 360°, so there are 12 sides.

d. $180 - 30 = 150^\circ$ The interior and exterior angles lie on a straight line, so they add up to 180°. The interior angle is 150°.

Answer to Question 5

a. $1440 \div 180 = 8$, so $n = 8 + 2 =$ **10** The angle sum is 8 lots of 180°, so the polygon splits into 8 triangles and has 10 sides.

b. Exterior $= 180 - 140 = 40^\circ$. $360 \div 40 =$ **9** Interior and exterior angles add to 180°, and the exterior angles fill 360°, so the polygon has 9 sides.

c. $360 \div 15 =$ **24** The exterior angles add up to 360°, so the number of sides is $360 \div 15$: 24 sides.

d. $2340 \div 180 = 13$, so $n = 13 + 2 =$ **15** The sum is 13 lots of 180°, so the polygon has $13 + 2 = 15$ sides.

Answer to Question 6

a. Sum: $x + (x + 20) + 2x + (2x + 40) = 6x + 60 = 360$, so $6x = 300$ and $x = 50$. The angles are 50°**,** 70°**,** 100°**,** 140° (check: they add to 360). A quadrilateral's angles add up to 360°. Collect the x terms ($6x$) and the numbers (60), then solve the equation.

b. Each angle is $540 \div 5 = 108$. $3x + 6 = 108$, so $3x = 102$ and $x =$ **34**. A pentagon's angles add up to 540°, and in a regular pentagon they are all equal, so each is 108°. Check: $3 \times 34 + 6 = 108$.

c. Sum $= 720$. Known: 400. The other two share 320, so each is 160°. A hexagon's angles add up to 720°. The four angles of 100° make 400°, and $320 \div 2 = 160$, so the two equal angles are each 160°.

Answer to Question 7

a. Triangle 60°, square 90°, pentagon $540 \div 5 = 108^\circ$, hexagon $720 \div 6 = 120^\circ$, octagon $1080 \div 8 = 135^\circ$. Use $(n - 2) \times 180 \div n$ for each shape; the triangle and square angles are already known.

b. $3 \times 120 = 360$, so there is no gap. The angles at the point must add up to a full turn of 360°, and they do exactly, so hexagons tile.

c. $135 + 135 + 90 = 360$, so they fit. Two octagons and one square fill the full turn exactly, so this pattern also leaves no gap.

d. $360 \div 108 = 3.33\ldots$ is not a whole number. Three pentagons make 324°, leaving a gap of 36°; four would overlap. Pentagon angles cannot make exactly 360° at a

point, so regular pentagons alone cannot tile a floor.

e. $360 \div 30 = 12$ tiles along and $240 \div 30 = 8$ tiles across, so $12 \times 8 =$ **96 tiles**. Change the metres to centimetres first (360 cm and 240 cm), then count the rows and columns. Maya needs 96 tiles.

Answer to Question 8

a. $8 \times 30 =$ **240 cm** A regular octagon has 8 equal sides, so multiply the side length by 8. The perimeter is 240 cm.

b. Interior $= 135°$, exterior $= 360 \div 8 = 45°$. The exterior angle is $360 \div 8 = 45°$, so the interior angle is $180 - 45 = 135°$.

c. He turns through $8 \times 45 = 360°$ (one full turn). At each corner he turns through the exterior angle; after 8 corners he faces his starting direction again, a turn of $360°$.

d. $14 \times 42.50 =$ **£595** Multiply the price of one sign by the number of signs: the total cost is £595.

Answer to Question 9

a. $6 \times 6 =$ **36 mm** A hexagon has 6 equal sides of 6 mm, so the perimeter is 36 mm.

b. $720 \div 6 = 120°$ The angle sum of a hexagon is $720°$, shared between 6 equal angles, so each is $120°$.

c. $3 \times 120 = 360$, so the angles fill the full turn around a point. Three angles of $120°$ make exactly a full turn, so the cells fit together with no gaps or overlaps.

d. $3200 \times 9 =$ **28 800 cells** Multiply the number of cells in one frame by the number of frames: there are 28 800 cells in the hive.

e. Yes. Interior angle $108°$ does not divide 360, so pentagons would leave gaps. She is right, because $360 \div 108$ is not a whole number, so pentagons would leave gaps between the cells.

Answer to Question 10

a. Pentagon $108°$, hexagon $120°$. A pentagon's angle sum is $540°$ and a hexagon's is $720°$; divide by 5 and by 6.

b. $108 + 120 + 120 = 348°$ One pentagon angle and two hexagon angles meet at each corner, giving a total of $348°$.

c. It is $12°$ less than $360°$, so the shapes cannot lie flat. The gap makes the surface curve into a ball. On a flat surface the angles at a point must add up to $360°$. The missing $12°$ forces the panels to bend, which is why the football is round.

d. Sides: $12 \times 5 + 20 \times 6 = 60 + 120 = 180$. Each edge is counted twice, so edges $= 180 \div 2 =$ **90**. Every edge is shared by two panels, so counting the sides of every panel counts each edge twice. The ball has 90 edges.

Answer to Question 11

a. Perimeter $= 5 \times 2 = 10$ m, cost $= 10 \times 12 =$ **£120** The fence goes all the way round, so first find the perimeter, then multiply by the cost per metre. The fence costs £120.
b. 108° A regular pentagon has an angle sum of 540°, so each interior angle is $540 \div 5 = 108^\circ$.
c. Angle $B = 108^\circ$, so each base angle is $(180 - 108) \div 2 = 36^\circ$ The angles in triangle ABC add up to 180°, and the two base angles are equal because $AB = BC$, so angle $BAC = 36^\circ$.
d. Angle $CAE = 108 - 36 = 72^\circ$ Angle BAE is split into angle BAC and angle CAE, so subtract: angle $CAE = 72^\circ$.

Answer to Question 12

a. $360 \div 40 =$ **9** The robot turns through the exterior angle each time, and the turns add up to 360°, so it repeats the command 9 times.
b. A regular **nonagon** (9 sides) The path has 9 equal sides and 9 equal angles, so it is a regular nonagon.
c. $9 \times 2 =$ **18 m** Each of the 9 sides is 2 m long, so the robot travels 18 m.
d. $360 \div 72 = 5$, so a regular **pentagon**, with interior angle $180 - 72 = 108^\circ$. Now the exterior angle is 72°, giving 5 sides, so the robot draws a regular pentagon with interior angles of 108°.

Answer to Question 13

a. Sum $= 540$. Known: $98 + 121 + 104 + 117 = 440$. Fifth $= 540 - 440 = 100^\circ$ A pentagon's angles add up to 540°. Subtract the four measured angles: the fifth angle is 100°.
b. $40 + 55 + 62 + 48 + 51 =$ **256 m** The perimeter is the total of the five side lengths: 256 m.
c. $256 \times 7.50 =$ **£1 920** Multiply the perimeter by the cost per metre: fencing the whole field costs £1 920.
d. $252 \times 7.50 =$ **£1 890** The gate gap means 4 m less fence: $256 - 4 = 252$ m. The new cost is £1 890, which is £30 less.

Answer to Question 14

a. $360 \div 12 = 30^\circ$ The exterior angles of any polygon add up to 360°, and there are 12 equal ones, so each is 30°.
b. $180 - 30 = 150^\circ$ The interior and exterior angles lie on a straight line, so the interior angle is 150°.
c. The twelve apex angles fill a full turn, so $360 \div 12 = 30^\circ$. The twelve apex angles

meet at the centre and fill a full turn of 360°, so each is 30°.

d. $(180 - 30) \div 2 = 75^\circ$ each. Two base angles meet at each vertex: $75 + 75 = 150$, which equals the interior angle. The angles in each triangle add up to 180°, so each base angle is 75°. The check confirms the interior angle of 150°.

Answer to Question 15

a. Pentagon: sum 540°, exterior 72°. Hexagon: sum 720°, interior 120°. Octagon: interior 135°, exterior 45°. Decagon: exterior 36°. Last row: $360 \div 24 = 15$ sides, a 15-gon with sum $13 \times 180 = 2340^\circ$ and interior 156°. Use the angle sum $(n - 2) \times 180$, the interior angle (sum divided by n) and the exterior angle ($360 \div n$) in each row.

b. Interior $= 2\times$ exterior and they add to 180, so $3e = 180$, $e = 60$, $n = 360 \div 60 = 6$: a **hexagon**. Let the exterior angle be e. Interior + exterior $= 180$, so $2e + e = 180$. The polygon is a regular hexagon.

c. $6e = 180$, so $e = 30$ and $n = 360 \div 30 = 12$: a **dodecagon**. This time $5e + e = 180$, so $e = 30$. A regular polygon with 12 sides is a dodecagon.

d. Exterior 20° gives $n = 360 \div 20 = 18$ sides, so such a polygon has 18 sides, not fewer than 15. (The two angles do add to 180°, so it is consistent.) An exterior angle of 20° forces exactly 18 sides, so the claim about having fewer than 15 sides is impossible.

Answer to Question 16

a. 108° The pentagon is regular, so every interior angle is $540 \div 5 = 108^\circ$.

b. $(180 - 108) \div 2 = 36^\circ$ each. Triangle ABC has $AB = BC$, so the base angles are equal, and the angles in a triangle add up to 180°.

c. Angle $EAD = 36^\circ$ by congruence, so $CAD = 108 - 36 - 36 = 36^\circ$. Triangles ABC and AED are congruent, so angle EAD also measures 36°; subtract both from 108°.

d. $36 + 36 + 36 = 108$. The three angles at A are each 36°, so together they make the full interior angle of 108°. The diagonals split the angle into three equal parts.

e. Angles in triangle ACD: $180 - 36 = 144$, shared equally: 72°. In triangle ACD, angle $CAD = 36^\circ$, so the other two equal angles share 144°. Angle $ACD = 72^\circ$.

Answer to Question 17

a. Only **Ali** is correct: $5 \times 180 = 900$. Use $(n - 2) \times 180$ to test Ali, $360\div$ exterior angle to test Bea and Cal, and divide by 180 to test Dev.

b. Bea: exterior angle would be 80° and $360 \div 80 = 4.5$ is not a whole number, so no regular polygon has interior angle 100°. Cal: it is $360 \div 20 = 18^\circ$. Dev: $1000 \div 180 = 5.56$, which is not a whole number, so no polygon has this angle sum (the sum is always a multiple of 180). For a claim to be true, the number of

sides must be a whole number of at least 3. Only Ali's claim passes this test.

c. The smallest polygon has 3 sides, so the largest exterior angle is $360 \div 3 = 120^\circ$: the equilateral triangle. The fewer sides a regular polygon has, the bigger each exterior angle is. A triangle has the fewest sides, so its exterior angle of 120° is the largest.

Answer to Question 18

a. $1080 \div 8 = 135^\circ$ An octagon has an angle sum of $6 \times 180 = 1080^\circ$, shared equally between 8 angles, so each is 135°.

b. $135 \div 2 = 67.5^\circ$ Each rail is cut at half the interior angle, so the two cut ends fit together to make 135°. The cut angle is 67.5°.

c. $8 \times 1.5 =$ **12 m** The frame has 8 equal sides of 1.5 m, so the perimeter is 12 m.

d. $12 \div 0.5 =$ **24 posts** (it is a closed loop, so the number of posts equals the number of gaps). Divide the perimeter by the spacing. Around a closed shape the first and last posts are not doubled, so 24 posts are needed.

e. $24 \times 3.50 =$ **£84** Multiply the number of posts by the price of one post: the posts cost £84.

Answer to Question 19

a. $5 \times 180 = 900^\circ$ A heptagon has 7 sides, so it splits into 5 triangles: the angles add up to 900°.

b. Interior $= 900 \div 7 = 128.6^\circ$; exterior $= 360 \div 7 = 51.4^\circ$. Share the angle sum between 7 equal angles, and divide 360° by 7 for the exterior angle. Check: $128.6 + 51.4 = 180$.

c. $7 \times 2.4 =$ **16.8 cm** The coin has 7 equal sides of 2.4 cm, so the perimeter is 16.8 cm.

d. $360 \div 128.57 = 2.8$ is not a whole number: two coins leave a gap and three would overlap. For shapes to fit around a point with no gaps, the interior angles must divide exactly into 360°. They do not, so seven heptagons cannot fit.

e. Two coins use $2 \times 128.57 = 257.1^\circ$, so the gap is $360 - 257.1 = 102.9^\circ$. Two heptagon angles leave a space of about 102.9°, which is too small for a third coin.

Answer to Question 20

a. Sum $= (n - 2) \times 180$. An n-sided polygon splits into $n - 2$ triangles, each with angles adding up to 180°.

b. Sum $= 150(n - 1) + 90$. There are $n - 1$ angles of 150° and one angle of 90°.

c. $180n - 360 = 150n - 150 + 90 = 150n - 60$. So $30n = 300$ and $n =$ **10**. The two expressions give the same angle sum, so set them equal, expand the brackets and collect the n terms. The polygon has 10 sides.

d. Sum $= 8 \times 180 = 1440$. Also $9 \times 150 + 90 = 1350 + 90 = 1440$. They agree. Both

methods give 1440°, so the answer $n = 10$ is correct.