# Answers - Volume and surface area of prisms

Year: Year 8

## Answer to Question 1

Volume of a cuboid = length × width × height. Surface area = 2 × (sum of the areas of the three different faces).

a. $V = 8 \times 5 \times 4 =$ **160 cm³**. $SA = 2(40 + 32 + 20) =$ **184 cm²**.
b. $V = 6^3 =$ **216 cm³**. $SA = 6 \times 36 =$ **216 cm²**.
c. $V = 2.5 \times 1.2 \times 0.8 =$ **2.4 m³**. $SA = 2(3 + 2 + 0.96) =$ **11.92 m²**.
d. Side $= \sqrt[3]{125} = 5$ cm. $SA = 6 \times 25 =$ **150 cm²**.

## Answer to Question 2

First find the area of the cross-section using the right formula for its shape, then multiply by the length of the prism.

a. Area $= \frac{1}{2} \times 6 \times 4 = 12$ cm². $V = 12 \times 10 =$ **120 cm³**.
b. Area $= \frac{1}{2} \times 3 \times 4 = 6$ cm². $V = 6 \times 12 =$ **72 cm³**.
c. Area $= \frac{1}{2}(5 + 9) \times 4 = 28$ cm². $V = 28 \times 15 =$ **420 cm³**.
d. $360 \div 24 =$ **15 cm**

In (d) we work backwards: length = volume ÷ cross-section area.

## Answer to Question 3

a. $V = \pi r^2 h = \pi \times 5^2 \times 12 = 300\pi =$**942.5 cm³**.
b. Curved surface $= 2\pi rh = 2\pi \times 5 \times 12 = 120\pi =$**377.0 cm²**.
c. Two circular ends $2 \times \pi \times 5^2 = 50\pi$. Total $50\pi + 120\pi = 170\pi =$**534.1 cm²**.
d. Radius $8 \div 2 = 4$. $h = \dfrac{603}{\pi \times 4^2} = \dfrac{603}{50.27} =$**12.0 cm**.

The total surface area of a closed cylinder is always the two circles plus the curved rectangle, which unrolls to a width of $2\pi r$ and a length of h.

## Answer to Question 4

a. $\frac{1}{2} \times 5 \times 12 =$ **30 cm²**
b. $5 \times 20 = 100$, $12 \times 20 = 240$, $13 \times 20 = 260$ cm².
c. Two ends $= 60$. Rectangles $= 100 + 240 + 260 = 600$. Total = **660 cm²**.
d. $30 \times 20 =$ **600 cm³**

The three rectangles together have area $(5 + 12 + 13) \times 20 = 600$ cm², the perimeter of the triangle times the length.

## Answer to Question 5

a. $50 \times 40 \times 30 = 60000$ cm³. Divide by 1 000: **60 litres**.
b. 1 m³ holds 1 000 litres: $120 \times 1000 =$**120 000 litres**.
c. 1 litre = 1 000 cm³: $1.5 \times 1000 =$**1 500 cm³**.
d. $20^3 = 8000$ cm³, which is **8 litres**.

Converting the volume to cm³ first and then dividing by 1 000 is the safest route to litres.

## Answer to Question 6

a. $10 \times 6 \times h = 360$, so $h = 360 \div 60 =$**6 cm**.
b. Cross-section = volume ÷ length: $150 \div 10 =$**15 cm²**.
c. Triangle area $= 252 \div 12 = 21$ cm². Then $\frac{1}{2} \times 7 \times h = 21$, so $3.5h = 21$ and $h = 6$ **cm**.
d. $h = \frac{500}{\pi \times 4^2} = \frac{500}{50.27} = 9.95\ldots$, so **9.9 cm**.

Each part reverses the volume formula: divide the volume by everything you already know.

## Answer to Question 7

a. Depth of water $= 40$ cm. $V = 80 \times 35 \times 40 =$ **112 000 cm³**
b. **112 litres**
c. **112 kg**
d. Base $80 \times 35 = 2800$. Long sides $2 \times 80 \times 45 = 7200$. Short sides $2 \times 35 \times 45 = 3150$. Total $=$ **13 150 cm²**.
e. $13150 \times 0.012 =$ **£157.80**

Only the water depth (40 cm) is used for the water volume, but the full height (45 cm) is used for the glass.

## Answer to Question 8

a. $2(2.4 \times 2 + 1.8 \times 2) = 2 \times 8.4 =$ **16.8 m²**
b. $2.4 \times 1.8 =$ **4.32 m²**
c. Total $= 21.12$. Door $= 1.44$, window $= 0.6$. Paint area $=$ **19.08 m²**.
d. $19.08 \div 6 = 3.18$, so **4 litres**.

e. $4 \times 14.50 =$ **£58**

Paint must be bought in whole litres, so 3.18 litres rounds up to 4 litres.

## Answer to Question 9

a. Area of triangle $= \frac{1}{2} \times 2 \times 1.5 = 1.5$ m². $V = 1.5 \times 3 =$ **4.5 m³**
b. $\sqrt{1^2 + 1.5^2} = \sqrt{3.25} =$ **1.80 m**
c. $2 \times 1.80 \times 3 =$ **10.8 m²**
d. Floor $= 2 \times 3 = 6$. Ends $= 2 \times 1.5 = 3$. Total $= 10.8 + 6 + 3 =$ **19.8 m²**.

The sloping side uses Pythagoras in the right-angled triangle formed by half the base (1 m) and the height (1.5 m).

## Answer to Question 10

a. There are 100 cm in a metre: $8 \div 100 =$**0.08 m**.
b. All lengths in metres: $12 \times 1.2 \times 0.08 =$**1.152 m³**.
c. $1.152 \times 95 =$**£109.44**.
d. 1.152 rounds up to the next quarter, **1.25 m³**, costing $1.25 \times 95 =$**£118.75**.

## Answer to Question 11

a. Radius $7.4 \div 2 = 3.7$ cm. $V = \pi \times 3.7^2 \times 10.8 = 464.5$, about **464 cm³**.
b. 1 cm³ = 1 ml, so **464 ml**.
c. The label is the curved surface: $\pi \times 7.4 \times 10.8 = 251.1$, about **251 cm²**.
d. $25 \times 12 = 300$ tins. $300 \times 464 = 139200$ ml, about **139 litres**.

## Answer to Question 12

a. $\frac{1}{2} \times 3.5 \times 3 = 5.25$ cm². $V = 5.25 \times 20 =$ **105 cm³**
b. $3 \times 3 \times 20 =$ **180 cm³**
c. The cuboid holds more. Difference $= 75$ cm³ and $\frac{75}{180} =$ **41.7%** of the cuboid.
d. $105 \times 0.9 = 94.5$p, so about **95p**.

Both bars have the same length, so comparing the cross-section areas (5.25 cm² and 9 cm²) gives the same conclusion.

## Answer to Question 13

a. $\frac{1}{2}(40 + 30) \times 25 =$ **875 cm²**

b. $875 \times 200 = 175000$ cm³ = **175 litres**
c. $175 \div 10 = 17.5$ min = **17 min 30 s**
d. $175 \div 60 = 2.9$, so **2 cows** for a full day.

Remember to change 2 m into 200 cm before multiplying, so that all lengths are in centimetres.

## Answer to Question 14

a. Radius = half the diameter: $2.4 \div 2 =$**1.2 m**.
b. $V = \pi \times 1.2^2 \times 0.25 = 1.131$, so **1.13 m³**.
c. $1.131 \div 0.04 = 28.3$. 28 bags are not quite enough, so **29 bags**.
d. $29 \times 3.20 =$**£92.80**.

Working in metres throughout keeps the volume in m³, the same unit as the bags.

## Answer to Question 15

a. Whole $12 \times 10 = 120$ minus the missing corner $6 \times 5 = 30$: **90 cm²**.
b. $90 \times 20 =$ **1800 cm³**
c. Perimeter $= 12 + 10 + 6 + 5 + 6 + 5 =$ **44 cm**.
d. Two ends $= 180$. Sides $= 44 \times 20 = 880$. Total = **1060 cm²**.

The perimeter of the L-shape is the same as that of the full 12 by 10 rectangle, because the cut-out corner replaces two edges with two equal edges.

## Answer to Question 16

a. A: $10 \times 7 \times 20 =$**1 400 cm³**. B: radius 4 cm, $\pi \times 4^2 \times 20 = 320\pi =$**1 005 cm³**.
b. **Container A** holds more, by $1400 - 1005 =$**395 ml**.
c. A: $2(10 \times 7 + 10 \times 20 + 7 \times 20) = 2(70 + 200 + 140) =$**820 cm²**. B: ends $2 \times 16\pi = 32\pi$, curved $2\pi \times 4 \times 20 = 160\pi$; total $192\pi =$**603 cm²**.
d. A: $820 \times 0.4 = 328$p. B: $603 \times 0.4 = 241$p. **Container B** is cheaper to make, by 87p.
e. Cardboard cost per ml: A $328 \div 1400 \approx 0.234$p, B $241 \div 1005 \approx 0.240$p. **Container A** is slightly better value, because it holds much more juice for its extra cardboard.

## Answer to Question 17

a. $\frac{1}{2}(1 + 2.5) \times 20 =$ **35 m²**
b. $35 \times 10 =$ **350 m³**
c. $350 \times 1000 =$ **350 000 litres**
d. $350000 \div 6000 = 58.3$ min, about **58 min 20 s**, so just under 1 hour.

e. Floor length $= \sqrt{20^2 + 1.5^2} = 20.06$ m, floor area $= 200.6$ m². Two side walls $= 2 \times 35 = 70$. End walls $= 10 \times 1 + 10 \times 2.5 = 35$. Total $\approx 200.6 + 70 + 35 = 305.6$ m², cost ≈ **£6 723**.

The trapezium cross-section is the side view of the pool, so the volume is that area times the 10 m width.

## Answer to Question 18

a. $12 \times 9 \times 5 =$**540 cm³**; melting does not change the volume.
b. $\pi \times 3^2 \times h = 540$, so $h = \dfrac{540}{9\pi} = 19.09\ldots$, **19.1 cm**.
c. $2\pi \times 3 \times 19.1 = 360.0$, about **360 cm²**.
d. Mass = density × volume: $540 \times 7.8 = 4212$ g, which is **4.212 kg**.

## Answer to Question 19

a. Area $= \frac{1}{2} \times 8 \times 5 = 20$ cm². $V = 20 \times 12 =$ **240 cm³**.
b. **Ali**
c. Bea forgot the $\frac{1}{2}$. Cal added instead of multiplying $(8 + 5 + 12)$ or used the wrong numbers. Dev found only the triangle area times 3 (60), not the length.

The volume of a triangular prism must include the half in the triangle's area; checking that a prism is half of a 8 by 5 by 12 cuboid (480 cm³) confirms 240 cm³.

## Answer to Question 20

a. $r = 3$: $h = 17.7$, SA $= 2\pi \times 9 + 2\pi \times 3 \times 17.7 =$**389.9**. $r = 4$: $h = 9.9$, SA **350.5**. $r = 5$: $h = 6.4$, SA **357.1**. $r = 6$: $h = 4.4$, SA **392.9** (cm and cm²).
b. $r = 4$ gives the smallest surface area in the table; trying more radii shows the true minimum is at about **4.3 cm**.
c. At $r \approx 4.3$ the height is about 8.6 cm, equal to the diameter: the best tin is **as tall as it is wide** ($h \approx 2r$).
d. Tins are also chosen for stacking, gripping, shelf space and label size, so the shape using the least metal is not always the most practical.