Worksheet - Transformations

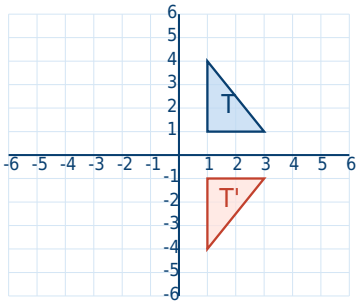
Year: Year 8 · 20

Show your working.

Question 1

Question 1 – Reflecting points and shapes *(Section A · Skills practice)*

Triangle T has vertices $A(1,1)$, $B(3,1)$ and $C(1,4)$.



a. Write the coordinates of the image of T after reflection in the x-axis.
b. Write the coordinates after reflection in the y-axis.
c. Write the coordinates after reflection in the line $y = x$.
d. Reflect point $A(1,1)$ in the line $x = 3$.

Question 2

Question 2 – Translation *(Section A · Skills practice)*

Translate triangle T with vertices $(1,1)$, $(3,1)$, $(1,4)$ by each vector. Write the new coordinates.

a. $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$
b. $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$
c. $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$
d. Point $P(2,3)$ is mapped to $P'(-1,7)$. Find the translation vector.

Question 3

Question 3 – Rotation *(Section A · Skills practice)*

Rotate triangle T with vertices $(1,1)$, $(3,1)$, $(1,4)$ about the origin O.

a. 90° anticlockwise

b. 180°
c. 90° clockwise
d. Describe fully the rotation that maps $(3, 1)$ to $(-1, 3)$.

Question 4

Question 4 – Enlargement from the origin *(Section A · Skills practice)*

Enlarge triangle T with vertices $(1, 1)$, $(3, 1)$, $(1, 4)$ from the origin.

a. Scale factor 2
b. Scale factor 3
c. Scale factor $\frac{1}{2}$ (use $(2, 2)$, $(6, 2)$, $(2, 8)$ as the original instead)
d. State the new side lengths for scale factor 2 if the original base is 2 units.

Question 5

Question 5 – Scale factor, perimeter and area *(Section A · Skills practice)*

Rectangle R is 4 cm by 3 cm. It is enlarged by scale factor 3.

a. Find the dimensions of the enlarged rectangle.
b. Find the perimeters of R and the image. What is the ratio?
c. Find the areas of R and the image. What is the ratio?
d. A shape of area 10 cm² is enlarged by scale factor 4. Find the new area.

Question 6

Question 6 – Describing a transformation *(Section A · Skills practice)*

Give the full description of each transformation that maps $\triangle P$ to $\triangle Q$.

a. P has vertices $(1, 2)$, $(4, 2)$, $(1, 5)$. Q has vertices $(-4, 2)$, $(-1, 2)$, $(-4, 5)$.
b. P has vertices $(1, 2)$, $(3, 2)$, $(1, 3)$. Q has vertices $(1, -2)$, $(3, -2)$, $(1, -3)$.
c. P has vertices $(1, 1)$, $(2, 1)$, $(1, 3)$. Q has vertices $(2, 2)$, $(4, 2)$, $(2, 6)$.
d. Which of the three pairs of triangles are congruent and which are similar?

Question 7

Question 7 – Tiling a bathroom wall *(Section B · Real-life practice)*

Maya is tiling a bathroom wall with a pattern. One tile has a corner at $(0, 0)$ and measures 1 unit by 1 unit on a coordinate grid, where 1 unit is 20 cm.

a. The pattern repeats by translation. A tile is moved by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Write the translation for the tile above it.
b. Write the translation that maps the first tile to the tile in column 5, row 3

(counting from 1).
c. How many tiles are needed to cover a wall 3 m wide and 2.4 m high?
d. Tiles cost £1.85 each and are sold in boxes of 10 for £16. Which is cheaper for the whole wall, and by how much?

Question 8

Question 8 – A butterfly logo *(Section B · Real-life practice)*

A design company draws half of a butterfly on a grid. The left wing has corners at $(-1, 1)$, $(-4, 3)$, $(-5, 0)$ and $(-3, -2)$. The y-axis is the line of symmetry.

a. Reflect each corner in the y-axis to complete the right wing.
b. What is the equation of the mirror line?
c. The logo is enlarged by scale factor 2.5 for a poster. How wide is the poster logo if the original is 10 cm wide?
d. The original has area 24 cm². What is the area on the poster?

Question 9

Question 9 – The Ferris wheel *(Section B · Real-life practice)*

The London Eye wheel turns anticlockwise. A capsule is at the position $(0, 60)$ relative to the centre at $O(0, 0)$ with radius 60 m, measured in metres.

a. Where is the capsule after a rotation of 90° anticlockwise about O?
b. Where is it after 180°?
c. Where is it after 270°?
d. One full turn takes 30 minutes. How long does a 90° turn take?

Question 10

Question 10 – Photocopying a worksheet *(Section B · Real-life practice)*

A photocopier enlarges a worksheet by 141% (scale factor 1.41) to change A4 to A3. An A4 page is 21 cm by 29.7 cm.

a. Find the dimensions of the A3 copy to 1 decimal place.
b. The original has a picture of area 50 cm². What is the area of the picture on the copy?
c. The copy is then reduced by scale factor 0.71. Find the overall scale factor and say what it does to the page.
d. A pupil says "enlarging by 141% doubles the area". Check this claim.

Question 11

Question 11 – Model railway *(Section B · Real-life practice)*

A model railway has a scale factor of $\frac{1}{76}$ compared with real trains.

a. A real locomotive is 19 m long. How long is the model in cm to 1 decimal place?
b. A model carriage is 25 cm long. How long is the real carriage in metres to 1 decimal place?
c. The real train has a window of area 0.9 m². Find the area of the model window in cm².
d. The model tunnel is 40 cm long. How many metres is the real tunnel?

Question 12

Question 12 – Shadows and similar shapes *(Section B · Real-life practice)*

A lamp post casts a shadow of 4 m while a 1.5 m tall child casts a shadow of 1 m at the same time. The triangles formed by each person and their shadow are similar.

a. What is the scale factor from the child's triangle to the lamp post's triangle?
b. Find the height of the lamp post.
c. The tree nearby has a shadow of 6.4 m. Find its height.
d. At a different time the child's shadow is 2 m. How long is the lamp post's shadow?

Question 13

Question 13 – Moving furniture *(Section B · Real-life practice)*

A plan of a classroom is drawn on a grid where 1 square is 1 m. A desk is a rectangle with corners at $(2, 1)$, $(4, 1)$, $(4, 2)$ and $(2, 2)$.

a. The desk is moved by $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$. Write its new corners.
b. The teacher wants the desk moved to have corner $(9, 5)$ in place of $(4, 2)$. Is this the same translation?
c. The desk is rotated 90° clockwise about $(2, 1)$. Where does corner $(4, 1)$ go?
d. What is the area of the desk before and after rotation?

Question 14

Question 14 – Enlarging a garden plan *(Section B · Real-life practice)*

An architect draws a plan of a rectangular garden 8 cm by 5 cm. The scale is 1 cm to 2 m, so the real garden is an enlargement of the plan with scale factor 200 (as 1 cm to 2 m = 1 : 200).

a. Find the real length and width in metres.
b. Find the real area in m².
c. A pond on the plan has an area of 3 cm². Find the real area of the pond in m².
d. The owner wants a copy of the plan at twice the size. What is the scale of the new plan, as a ratio?

Question 15

Question 15 – Combining transformations *(Section C · Problem solving)*

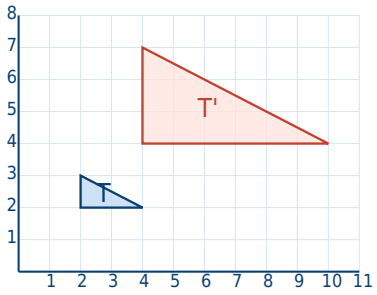
Triangle T has vertices $(1, 1)$, $(3, 1)$ and $(1, 4)$. T is reflected in the y-axis to give T_1, and then T_1 is translated by $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$ to give T_2.

a. Write the coordinates of T_1.
b. Write the coordinates of T_2.
c. Describe the single transformation that maps T to T_2 if possible, or explain why it cannot be done by one translation.
d. Is T_2 congruent to T? Explain.

Question 16

Question 16 – Enlargement from another centre *(Section C · Problem solving)*

Triangle $A(2, 2)$, $B(4, 2)$, $C(2, 3)$ is enlarged by scale factor 3 with centre $(1, 1)$.



a. Find the image of A.
b. Find the image of B and C.
c. Compare the side lengths of the image and the object.
d. Find the area of the object and of the image.

Question 17

Question 17 – Pattern of squares *(Section C · Problem solving)*

A square pattern is made by enlarging a unit square by scale factors 1, 2, 3, 4 from a corner.

Scale factor k	1	2	3	4
Side (cm)	2	?	?	?
Area (cm²)	4	?	?	?

a. Complete the table.
b. What is the pattern in the areas? Write the area in terms of k.
c. Which scale factor gives a square of area 144 cm²?
d. Compare the area when $k = 4$ with the area when $k = 2$. What do you notice?

Question 18

Question 18 – Symmetry and transformations *(Section C · Problem solving)*

A shape has vertices $(1, 0)$, $(2, 3)$, $(0, 5)$ and $(-1, 3)$.

a. Reflect the shape in the x-axis and write the new vertices.
b. Reflect your image in the y-axis. What single transformation maps the original to the final image?
c. Is the shape symmetrical about the y-axis? Explain.
d. What is the equation of the line of symmetry of the combined shape of the original and its reflection in the x-axis?

Question 19

Question 19 – Spot the mistake *(Section C · Problem solving)*

Four pupils describe the same transformation of a triangle from $(1, 1), (2, 1), (1, 3)$ to $(-1, 1), (-2, 1), (-1, 3)$.

Pupil	Description
Ali	Reflection in the y-axis
Bea	Translation by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$
Cal	Rotation 180° about the origin
Dev	Reflection in the x-axis

a. Which pupil is correct?
b. Check why the other descriptions are wrong by finding the image of $(1, 1)$ in each case.
c. Find the image of $(2, 1)$ under Bea's translation and compare it with the correct image.

Question 20

Question 20 – Challenge: negative scale factor *(Section D · Challenge)*

Triangle P has vertices $(1, 1)$, $(3, 1)$ and $(1, 2)$. It is enlarged with scale factor -2 and centre the origin.

a. Find the image of each vertex.
b. Describe how the image differs from an enlargement with scale factor $+2$.
c. Find the area of the image.
d. A negative scale factor is the same as a positive enlargement followed by what other transformation?