Answers - Transformations

Year: Year 8

Answer to Question 1

a. $(x, y) \to (x, -y)$: $(1, -1)$, $(3, -1)$, $(1, -4)$
b. $(x, y) \to (-x, y)$: $(-1, 1)$, $(-3, 1)$, $(-1, 4)$
c. $(x, y) \to (y, x)$: $(1, 1)$, $(1, 3)$, $(4, 1)$
d. A is 2 to the left of the line $x = 3$, so the image is 2 to the right: $(5, 1)$

Each reflection keeps the triangle the same size and shape; only its position and orientation change.

Answer to Question 2

A translation by $\begin{pmatrix} a \\ b \end{pmatrix}$ adds a to every x-coordinate and b to every y-coordinate: positive a moves right, positive b moves up.

a. Add 4 to x and 2 to y: $(5, 3)$, $(7, 3)$, $(5, 6)$
b. $(-2, 2)$, $(0, 2)$, $(-2, 5)$
c. $(1, -4)$, $(3, -4)$, $(1, -1)$
d. $-1 - 2 = -3$ and $7 - 3 = 4$: $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

Answer to Question 3

Rotations about the origin follow simple rules: 90° anticlockwise $(x, y) \to (-y, x)$, 180° $(x, y) \to (-x, -y)$, 90° clockwise $(x, y) \to (y, -x)$.

a. $(x, y) \to (-y, x)$: $(-1, 1)$, $(-1, 3)$, $(-4, 1)$
b. $(x, y) \to (-x, -y)$: $(-1, -1)$, $(-3, -1)$, $(-1, -4)$
c. $(x, y) \to (y, -x)$: $(1, -1)$, $(1, -3)$, $(4, -1)$
d. **Rotation 90° anticlockwise about the origin** ($(3, 1) \to (-1, 3)$).

A full description of a rotation needs three things: the angle, the direction and the centre.

Answer to Question 4

When the centre of enlargement is the origin, multiply every coordinate by the scale factor.

a. Multiply each coordinate by 2: $(2, 2)$, $(6, 2)$, $(2, 8)$
b. $(3, 3)$, $(9, 3)$, $(3, 12)$

c. Halve each coordinate: $(1, 1)$, $(3, 1)$, $(1, 4)$
d. Base $= 2 \times 2 =$ **4 units**.

A scale factor between 0 and 1 makes the shape smaller, which is still called an enlargement in maths.

Answer to Question 5

Multiply each length by the scale factor. Perimeters scale by the same factor as lengths, but areas scale by the square of the scale factor.

a. 12 cm by 9 cm.
b. Original 14 cm; image 42 cm; ratio $1 : 3$ (same as the scale factor).
c. Original 12 cm²; image 108 cm²; ratio $1 : 9$ (the scale factor squared).
d. $10 \times 4^2 =$ **160 cm²**

Check (c): $12 \times 9 = 108$, and $108 \div 12 = 9 = 3^2$.

Answer to Question 6

a. Every point moves 5 to the left: **translation by** $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$.
b. y changes sign: **reflection in the x-axis**.
c. Each coordinate doubles: **enlargement, scale factor 2, centre** $(0, 0)$.
d. (a) and (b) are **congruent** (same size). (c) is **similar** but not congruent.

Reflections, rotations and translations give congruent images; an enlargement with a scale factor other than 1 gives a similar image.

Answer to Question 7

a. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
b. Column 5 is 4 steps right and row 3 is 2 steps up: $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$
c. Width: $300 \div 20 = 15$ tiles. Height: $240 \div 20 = 12$ tiles. Total **180 tiles**.
d. Single tiles: $180 \times 1.85 = £333$. Boxes: 18 boxes $\times 16 = £288$. **Boxes** are cheaper by **£45**.

Always check that the number of tiles is a whole number in each direction: here 20 cm divides exactly into both 300 cm and 240 cm.

Answer to Question 8

a. Reflecting in the y-axis changes the sign of x: $(1, 1)$, $(4, 3)$, $(5, 0)$, $(3, -2)$.

b. The y-axis is the line where x is always 0: $x = 0$.
c. Lengths are multiplied by the scale factor: $10 \times 2.5 =$**25 cm**.
d. Areas are multiplied by the scale factor squared: $24 \times 2.5^2 = 24 \times 6.25 =$**150 cm²**.

Answer to Question 9

a. Using $(x, y) \to (-y, x)$: $(0, 60) \to (-60, 0)$, a quarter turn from the top to the left.
b. Half a turn: $(0, 60) \to (0, -60)$, at the bottom of the wheel.
c. Three quarters of a turn: $(0, 60) \to (60, 0)$, on the right.
d. 90° is a quarter of a full turn, so it takes $30 \div 4 =$**7.5 minutes**.

Answer to Question 10

a. $21 \times 1.41 =$ **29.6 cm** and $29.7 \times 1.41 =$ **41.9 cm** (A3 is 29.7 by 42).
b. $50 \times 1.41^2 = 50 \times 1.988 =$ **99.4 cm²**
c. $1.41 \times 0.71 = 1.0011 \approx 1$, so the page returns to **almost exactly its original size**.
d. $1.41^2 = 1.988 \approx 2$: **yes**, the area is almost doubled.

The scale factor 1.41 is close to $\sqrt{2}$, which is why A3 has twice the area of A4.

Answer to Question 11

a. 19 m = 1 900 cm. Divide by 76: $1900 \div 76 =$**25.0 cm**.
b. Multiply by 76: $25 \times 76 = 1900$ cm, which is **19.0 m**.
c. Areas scale by $76^2 = 5776$. 0.9 m² = 9 000 cm², so $9000 \div 5776 \approx$**1.6 cm²**.
d. $40 \times 76 = 3040$ cm, which is **30.4 m**.

Answer to Question 12

a. The lamp post's shadow is 4 times the child's: scale factor **4**.
b. Heights scale in the same way: $1.5 \times 4 =$**6 m**.
c. The tree's shadow is 6.4 times the child's, so its height is $1.5 \times 6.4 =$**9.6 m**.
d. The scale factor between child and lamp post is still 4: $2 \times 4 =$**8 m**.

Answer to Question 13

a. $(7, 4)$**,** $(9, 4)$**,** $(9, 5)$**,** $(7, 5)$

b. $(4, 2) \to (9, 5)$ is a move of $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$: **yes, the same**.
c. Relative to $(2, 1)$ the corner is $(2, 0)$; clockwise $90°$ gives $(0, -2)$, so the image is $(2, -1)$.
d. **2 m²** both before and after: rotation keeps area unchanged.

Answer to Question 14

a. $8 \times 2 =$ **16 m** and $5 \times 2 =$ **10 m**.
b. $16 \times 10 =$ **160 m²**
c. 1 cm² on the plan is $2 \times 2 = 4$ m², so $3 \times 4 =$ **12 m²**
d. Twice as big means 1 cm to 1 m, which is **1 : 100**.

Areas on the plan scale by the square of the length factor: 1 cm² stands for $2 \times 2 = 4$ m².

Answer to Question 15

a. T_1: $(-1, 1)$, $(-3, 1)$, $(-1, 4)$
b. T_2: $(-1, -4)$, $(-3, -4)$, $(-1, -1)$
c. No simple translation works, since the triangle is flipped (mirror image). The overall move is a **glide reflection**: reflect in the y-axis and translate down 5.
d. Yes: reflections and translations preserve length and angles, so T_2 is **congruent** to T.

Answer to Question 16

a. A is 1 right and 1 up from the centre. Image: 3 right, 3 up: $(4, 4)$
b. B: $(1 + 3 \times 3, 1 + 3 \times 1) = (10, 4)$. C: $(1 + 3 \times 1, 1 + 3 \times 2) = (4, 7)$.
c. Object base $AB = 2$, image base $= 6$; object height 1, image height 3. All lengths are **3 times** bigger.
d. Object: $\frac{1}{2} \times 2 \times 1 = 1$. Image: $\frac{1}{2} \times 6 \times 3 = 9$. Area is **9 times** bigger.

Answer to Question 17

a. Sides: 2, 4, 6, 8. Areas: 4, 16, 36, 64.
b. Area $= (2k)^2 = 4k^2$.
c. $4k^2 = 144$, $k^2 = 36$, $k = 6$.
d. Area at $k = 4$ is 64 and at $k = 2$ is 16: the ratio is **4**, which is $(4 \div 2)^2$.

Doubling the scale factor always multiplies the area by $2^2 = 4$.

Answer to Question 18

a. $(1, 0)$, $(2, -3)$, $(0, -5)$, $(-1, -3)$
b. Final image: $(-1, 0)$, $(-2, -3)$, $(0, -5)$, $(1, -3)$. Single transformation: **rotation** 180° **about the origin**.
c. The original is **not** symmetrical about the y-axis since $(1, 0)$ has no mirror point $(-1, 0)$.
d. $y = 0$ (the x-axis).

Reflecting in both axes, one after the other, always has the same effect as a half-turn about the origin: $(x, y) \to (-x, -y)$.

Answer to Question 19

a. **Ali** is correct: $(x, y) \to (-x, y)$ gives $(-1, 1)$, $(-2, 1)$, $(-1, 3)$ exactly.
b. Bea: $(1, 1) \to (-1, 1)$ works but $(2, 1) \to (0, 1)$, which is wrong. Cal: $(1, 1) \to (-1, -1)$, wrong. Dev: $(1, 1) \to (1, -1)$, wrong.
c. Bea's translation sends $(2, 1)$ to $(0, 1)$, but the correct image is $(-2, 1)$. A translation cannot flip a shape over, so it cannot match a reflection.

Answer to Question 20

a. Multiply each coordinate by -2: $(-2, -2)$, $(-6, -2)$, $(-2, -4)$
b. The image is on the **opposite side** of the centre and is upside down (rotated 180°).
c. Original area $= \frac{1}{2} \times 2 \times 1 = 1$. Image area $= 1 \times (-2)^2 = \mathbf{4}$.
d. A rotation of 180° about the centre.

Points on the image are 2 times as far from the centre as the object, but on the opposite side of it.