Lesson - Straight-line graphs

Year: Year 8

This lesson covers coordinates in four quadrants, drawing lines from equations, gradient, $y = mx + c$, parallel lines and where two lines meet.

Starter activity

A taxi driver charges a flat fee of £3 and then £2 for every kilometre. Draw a table for 0, 1, 2, 3, 4 and 5 km. Plot the points. What shape do they make? What is the cost when the distance is zero, and how much does the cost go up for each extra kilometre?

Coordinates in all four quadrants

Definition: coordinates and quadrants

A point is written (x, y): go along the x-axis first, then up or down the y-axis. The axes split the plane into four **quadrants**. In the top right both coordinates are positive; in the top left x is negative; in the bottom left both are negative; in the bottom right y is negative.

Rule: midpoint of a line segment

The **midpoint** of the line joining (x_1, y_1) and (x_2, y_2) is found by averaging: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. For $(2, 3)$ and $(8, 7)$, the midpoint is $(5, 5)$.

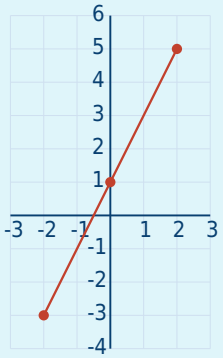
Equations of straight lines

Method: drawing a line from its equation

1. Make a table of values, choosing several values of x.
2. Work out y for each x.
3. Plot the points and join them with a ruler. If one point is off the line, recheck it.

Worked example 1: table of values

Draw the graph of $y = 2x + 1$ for x from -2 to 2.



When $x = -2$, $y = -3$. When $x = -1$, $y = -1$. When $x = 0$, $y = 1$. When $x = 1$, $y = 3$. When $x = 2$, $y = 5$. Plot $(-2, -3)$, $(-1, -1)$, $(0, 1)$, $(1, 3)$, $(2, 5)$ and join them.

A point lies on the line if its coordinates satisfy the equation. $(3, 7)$ lies on $y = 2x + 1$ because $2 \times 3 + 1 = 7$. $(2, 6)$ does not, because $2 \times 2 + 1 = 5 \neq 6$.

Key fact: special lines

- Horizontal lines have equations $y = k$. The line $y = 3$ passes through every point with y-coordinate 3.
- Vertical lines have equations $x = k$. The line $x = 4$ passes through $(4, 0)$, $(4, 5)$ and so on.

Gradient

Definition: gradient

The **gradient** measures how steep a line is. A line sloping up from left to right has a positive gradient; sloping down has a negative gradient. A horizontal line has gradient 0.

Rule: gradient

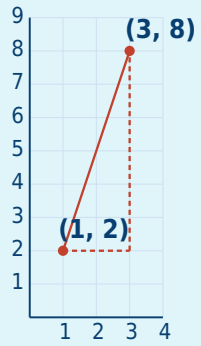
Pick two points on the line and use

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x}$$

Say "rise over run".

Worked example 2: finding a gradient

Find the gradient of the line through $(1, 2)$ and $(3, 8)$, and of the line through $(0, 5)$ and $(4, 1)$.



Change in $y = 8 - 2 = 6$. Change in $x = 3 - 1 = 2$. Gradient $= \frac{6}{2} = 3$. For $(0, 5)$ and $(4, 1)$: change in $y = 1 - 5 = -4$, change in $x = 4$, so the gradient is -1.

The form y = mx + c

Definition: gradient and y-intercept

When an equation is written $y = mx + c$, the number m is the gradient and c is the y-**intercept**, where the line crosses the y-axis. For $y = 3x - 4$, the gradient is 3 and the line crosses the y-axis at $(0, -4)$. For $y = -2x + 6$, $m = -2$ and $c = 6$.

Key fact: the x-intercept

To find where a line crosses the x-axis, set $y = 0$ and solve. For $y = 2x - 6$: $0 = 2x - 6$, so $x = 3$.

Worked example 3: rearranging into $y = mx + c$

Write $2y = 4x + 6$ and $x + y = 5$ in the form $y = mx + c$.

Divide every term by 2: $y = 2x + 3$, so the gradient is 2 and $c = 3$. For $x + y = 5$, subtract x to get $y = -x + 5$, so the gradient is -1.

Method: finding the equation of a line

1. Find the gradient m from two points.
2. Find c, the y-intercept, from the graph or by substituting a point.
3. Write $y = mx + c$ and check with a second point.

Worked example 4: finding the equation

Find the equation of the line through $(0, 2)$ and $(2, 8)$.

The gradient is $\frac{8 - 2}{2 - 0} = 3$. The line crosses the y-axis at $(0, 2)$, so $c = 2$. The equation is $y = 3x + 2$. Check with $(2, 8)$: $3 \times 2 + 2 = 8$.

Parallel lines and where lines meet

Rule: parallel lines

Parallel lines have the same gradient. The lines $y = 3x + 1$ and $y = 3x - 5$ are parallel, but $y = 2x + 1$ is not parallel to either of them.

Worked example 5: where two lines meet

Find where $y = 2x - 1$ and $y = x + 2$ cross.

The crossing point satisfies both equations, so set the two expressions for y equal: $2x - 1 = x + 2$, so $x = 3$ and $y = 5$. The lines meet at $(3, 5)$. From a graph, read off the coordinates of the crossing point.

Real-life graphs

Worked example 6: taxi fares

A taxi costs $C = 2m + 3$ pounds for m miles. Interpret the gradient and intercept and find the cost of 10 miles.

The gradient 2 is the cost per mile and the intercept 3 is the fixed starting charge. For 10 miles the cost is $2 \times 10 + 3 = £23$.

Watch out: common mistakes

- Writing the gradient as change in x over change in y.
- Subtracting coordinates in different orders on top and bottom.
- Reading c as the gradient. In $y = 3x - 4$, the gradient is 3 and the intercept is -4.
- Forgetting to divide every term when rearranging, as in $2y = 4x + 6$.
- Drawing a freehand curve through the points instead of using a ruler.

Worked example 7: equation of a line through two points

Find the equation of the line through $(1, 5)$ and $(4, 14)$.

Gradient $= \dfrac{14 - 5}{4 - 1} = \dfrac{9}{3} = 3$. So $y = 3x + c$. Substitute $(1, 5)$: $5 = 3 + c$, so $c = 2$. The line is $y = 3x + 2$. Check with the other point: $3 \times 4 + 2 = 14$. Correct.

Worked example 8: finding where a line crosses the axes

Find where $y = 2x - 8$ crosses the axes.

On the y-axis $x = 0$, so $y = -8$: the point is $(0, -8)$. On the x-axis $y = 0$, so $2x - 8 = 0$ and $x = 4$: the point is $(4, 0)$. Joining these two points draws the line.

Worked example 9: parallel line

Find the equation of the line parallel to $y = 3x - 1$ through $(2, 10)$.

Parallel lines have the same gradient, so $y = 3x + c$. Substitute: $10 = 6 + c$, so $c = 4$. The line is $y = 3x + 4$.

Worked example 10: rearranging into y = mx + c

Find the gradient and intercept of $4x + 2y = 10$.

Subtract $4x$: $2y = -4x + 10$. Divide by 2: $y = -2x + 5$. The gradient is -2 and the y-intercept is 5.

Note: the meaning of m and c

In real-life graphs the gradient is the *rate of change*, such as pounds per mile or litres per minute. The y-intercept is the *starting value*, such as a fixed fee or the amount at time zero. Write both with their units. A steeper line has a larger gradient, and a line that slopes down has a negative gradient.

Method: sketching a line quickly

1. Rearrange the equation into the form $y = mx + c$.
2. Plot the y-intercept $(0, c)$ on the vertical axis.
3. Use the gradient to move: for $m = \frac{3}{2}$ go 2 across and 3 up, and plot a second point.
4. Join the points with a ruler, extend the line across the grid, and label it with its equation.

Watch out: common mistakes

- Dividing the x change by the y change: gradient is rise over run, $\frac{\Delta y}{\Delta x}$.
- Mixing up the order of coordinates: (x, y), along first then up.
- Forgetting the sign: a line sloping downwards has a negative gradient.
- Reading $y = mx + c$ from an equation not yet rearranged, such as $2y = 6x + 8$, which has gradient 3 and not 6.

Did you know?

The coordinate system was named after the French philosopher René Descartes, who

is said to have had the idea while lying in bed watching a fly walk across the ceiling. He realised that its position could be described by two numbers: the distance from one wall and the distance from another. Today satellites, maps and video games all use coordinates in just the same way.

Reading a line in context

When a straight-line graph describes a real situation, its gradient and intercept always have a meaning, and explaining them is a common exam question. The gradient is a *rate*: it tells you how much the vertical quantity changes for each one unit of the horizontal quantity, such as pounds per mile, litres per minute or metres per hour. The intercept is the *starting value*: the value of the vertical quantity when the horizontal quantity is zero, such as a fixed charge or the amount in a tank at the start. Always give the units when you describe them, for example “the gradient 15 means the tank fills at 15 litres per minute”.

Example: from two points to a formula

A plumber charges £70 for a 1-hour job and £130 for a 3-hour job. Assuming a straight-line rule, find the formula for the charge C after h hours.

The gradient is the change in charge divided by the change in time: $\frac{130 - 70}{3 - 1} = 30$, so the plumber charges £30 per hour. Using $C = 30h + c$ with $h = 1$, $C = 70$: $70 = 30 + c$, so $c = 40$. The formula is $C = 30h + 40$: a £40 callout fee plus £30 per hour. Check with $h = 3$: $90 + 40 = 130$.

Did you know?

Road signs in the UK show the steepness of hills as a percentage gradient: a 10% hill rises 10 metres for every 100 metres travelled horizontally, which is a gradient of 0.1. Steeper signs warn drivers of very steep roads.

Otto's tip

Use a scale where each square is worth a sensible number, and label both axes. A gradient is a rise over a run, so say "rise over run" to yourself every time.