Answers - Straight-line graphs

Year: Year 8

Answer to Question 1

The midpoint is the mean of the x-coordinates and the mean of the y-coordinates; the length uses Pythagoras on the horizontal and vertical distances.

a. A is in the **second** quadrant (top left). B is in the **fourth** quadrant (bottom right).
b. Midpoint $= \left(\frac{-4+2}{2}, \frac{3+(-5)}{2}\right) = (-1, -1)$
c. Horizontal: $2-(-4) = 6$. Vertical: $3-(-5) = 8$.
d. $\sqrt{6^2+8^2} = \sqrt{100} =$ **10**

Answer to Question 2

Substitute each x value into the equation, taking care with negative numbers: for example, when $x = -2$, $2x+1 = -4+1 = -3$ and $3-x = 3+2 = 5$.

a. $y = -3, -1, 1, 3, 5$
b. $y = 5, 4, 3, 2, 1$
c. $y = 4, 1, 0, 1, 4$ (a U shape, a parabola)
d. $y = 2x+1$ and $y = 3-x$ are straight lines because x appears only to the power 1.

x	-2	-1	0	1	2
$y = 2x+1$	-3	-1	1	3	5
$y = 3-x$	5	4	3	2	1

Answer to Question 3

a. Change in y $= 11-2 = 9$; change in x $= 4-1 = 3$. Gradient $= 9 \div 3 =$ **3**.
b. Change in y $= -3-5 = -8$; change in x $= 2-(-2) = 4$. Gradient $= -8 \div 4 = -2$; the line slopes down.
c. Change in y is 0, so the gradient is **0**: the line is horizontal, $y = 4$.
d. $\frac{k-1}{8-0} = \frac{3}{4}$. Multiply by 8: $k-1 = 6$, so $k = 7$.

Always subtract the coordinates in the same order (second point minus first point) on the top and the bottom.

Answer to Question 4

In $y = mx + c$, m (the number multiplying x) is the gradient and c is the y-intercept. If

the equation is not in this form, rearrange it to make y the subject first.

a. Gradient **4**, y-intercept **-3**
b. Gradient $-\frac{1}{2}$, intercept **6**
c. Gradient **-5**, intercept **2**
d. $y = 3x + 4$: gradient **3**, intercept **4**
e. $y = -x + 7$: gradient **-1**, intercept **7**

Answer to Question 5

a. m = 3 and the line passes through $(0, -2)$, so c = −2: $y = 3x - 2$.
b. $y = -2x + c$. Substitute $(1, 5)$: $5 = -2 + c$, so $c = 7$: $y = -2x + 7$.
c. $m = \frac{7-1}{2-0} = 3$ and the intercept is 1: $y = 3x + 1$.
d. $m = \frac{10-4}{3-1} = 3$. Substitute $(1, 4)$: $4 = 3 + c$, $c = 1$: $y = 3x + 1$, the same line as (c).

Check each answer by substituting both points into the equation.

Answer to Question 6

Parallel lines have the same gradient. Lines of the form $y = k$ are horizontal and lines of the form $x = k$ are vertical.

a. $y = 2x - 5$ and $2y = 4x + 1$ (which is $y = 2x + 0.5$), gradient 2.
b. $y = 7$ is horizontal; $x = -2$ is vertical.
c. Same gradient -1 with $c = 6$: $y = -x + 6$
d. $m = 2$: $0 = 2 + c$, $c = -2$: $y = 2x - 2$

Check (d): $2 \times 1 - 2 = 0$, so the line passes through $(1, 0)$.

Answer to Question 7

a. $F(2) = 7$, $F(4) = 11$, $F(6) = 15$: correct.
b. Gradient **2**: £2 per mile. Intercept **3**: the fixed starting charge of £3.
c. $2 \times 9.5 + 3 =$ **£22**
d. $2m + 3 = 21$, $m =$ **9 miles**
e. $2m + 3 = 1.5m + 5$, $0.5m = 2$, $m = 4$ **miles** (fare £11).

In (e) the two fares are equal at 4 miles (£11); for longer journeys the second company is cheaper because its rate per mile is lower.

Answer to Question 8

a. $15 \times 4 + 60 =$ **120 litres**
b. $15t + 60 = 150$, $t =$ **6 minutes**
c. Gradient **15**: litres added per minute. Intercept **60**: the water in the tank at the start.
d. $15t + 60 = 300$, $t =$ **16 minutes**

Each part substitutes into $V = 15t + 60$ or solves $15t + 60 =$ a target volume by subtracting 60 and dividing by 15.

Answer to Question 9

a. $2 \times 5 + 8 =$ **£18**
b. $2d + 8 = 14$, $d =$ **3 GB**
c. 2 GB: A costs £12 and B costs £14, so **A**. 8 GB: A costs £24 and B costs £14, so **B**.
d. The gradient of plan B is **0**: the cost does not change with the data.

Plan A's line is steeper (gradient 2) and starts lower, so it is cheaper for small amounts of data and dearer for large amounts; the lines cross at 3 GB.

Answer to Question 10

a. $C = 0$: **32**. $C = 10$: **50**. $C = 100$: **212**.
b. Gradient **1.8**, intercept **32**.
c. $1.8 \times (-40) + 32 = -72 + 32 =$ **-40°F**: -40°C equals -40°F.
d. $C = 1.8C + 32 \Rightarrow -0.8C = 32 \Rightarrow C =$ **-40**

The gradient 1.8 means each degree Celsius is worth 1.8 degrees Fahrenheit; the intercept 32 is the freezing point of water in Fahrenheit.

Answer to Question 11

a. Substitute: $4 \times 1 + 4 = 8$, $4 \times 3 + 4 = 16$, $4 \times 5 + 4 = 24$. The formula matches the table.
b. $4 \times 7 + 4 = 28 + 4 =$**£32**.
c. $4h + 4 = 44$. Subtract 4: $4h = 40$, so $h = 10$ hours.
d. $4h + 4 = 5h$ gives $h = 4$ hours, where both cost **£20**. For longer hires the first shop is cheaper.

Answer to Question 12

a. Gradient **5**: he saves £5 per week. Intercept **12**: he started with £12.
b. $5w + 12 = 62$. Subtract 12: $5w = 50$, so $w = 10$ weeks.
c. Starting value 40 and rate 3: $L = 3w + 40$.
d. $5w + 12 = 3w + 40$. Subtract $3w$ and 12: $2w = 28$, $w = 14$. After **14 weeks** both

have **£82**.

Answer to Question 13

a. At $t = 0$, $h = 300$: she starts at **300 m**.
b. The gradient is **150 m per hour**.
c. Climbing 450 m needs $150t = 450$, so $t = 3$ **hours**; she is then at 750 m.
d. $150t + 300 = 1200$. Subtract 300: $150t = 900$, so $t = 6$ **hours**.

Answer to Question 14

a. Substitute: $1.5 \times 1 + 2 = 3.50$, $1.5 \times 2 + 2 = 5$, $1.5 \times 4 + 2 = 8$, $1.5 \times 6 + 2 = 11$. All correct.
b. $1.5 \times 5 + 2 = 7.5 + 2 =$**£9.50**.
c. $1.5h + 2 = 12.5$. Subtract 2: $1.5h = 10.5$, so $h = 7$ hours.
d. $1.5h + 2 = 2.5h$ gives $h = 2$: both cost **£5** for 2 hours; for longer stays the first car park is cheaper.

Answer to Question 15

a. Line 1: 1, 3, 5, 7. Line 2: 7, 6, 5, 4.
b. At $x = 2$ both give $y = 5$, so $P = (2, 5)$.
c. $3x = 6$, $x = 2$, $y = 5$. ✓
d. Example: Shop 1 charges £1 plus £2 per hour, shop 2 charges £7 minus £1 per hour. After 2 hours both cost £5.

Answer to Question 16

a. On the y-axis $x = 0$, so $y = 6$: $A(0, 6)$.
b. $0 = -1.5x + 6$ gives $1.5x = 6$, so $x = 4$: $B(4, 0)$.
c. OAB is right-angled at O with sides 4 and 6: $\frac{1}{2} \times 4 \times 6 =$**12 square units**.
d. Pythagoras: $\sqrt{4^2 + 6^2} = \sqrt{52} = 7.21\ldots$, so **7.2 units**.
e. Same gradient -1.5, through the origin so c = 0: $y = -1.5x$.

Answer to Question 17

a. Zara is **6 km** ahead.
b. $12t = 8t + 6$, $4t = 6$, $t =$ **1.5 hours**
c. $12 \times 1.5 =$ **18 km**

d. Tom: 36 km. Zara: 30 km. **Tom** is ahead by **6 km**.
e. Tom's gradient 12 means 12 km/h; Zara's 8 means 8 km/h; Tom is faster.

The intercepts show where each friend starts and the gradients show their speeds, so the faster rider (Tom) eventually overtakes.

Answer to Question 18

a. $\frac{13-7}{4-2} = \mathbf{3}$
b. $y = 3x + c$; $7 = 6 + c$, $c = 1$: $y = 3x + 1$
c. **Ali**
d. Bea: subtracted instead of adding to find c. Cal: used the reciprocal of the gradient. Dev: used the change in y (6) as the gradient and forgot to divide by the change in x.

Checking Ali's equation with both points: $3 \times 2 + 1 = 7$ and $3 \times 4 + 1 = 13$, so it passes through both.

Answer to Question 19

Each fare is a fixed charge (the intercept) plus a price per mile (the gradient).

a. A: $F = 1.5m + 4$. B: $F = 2m + 2.5$.
b. $1.5m + 4 = 2m + 2.5$, $1.5 = 0.5m$, $m = 3$ miles, fare **£8.50**.
c. A: $7.5 + 4 = £11.50$. B: $10 + 2.5 = £12.50$. **A** is cheaper.
d. After 3 miles, line B is steeper and above A, so **A is cheaper for longer journeys**; B is cheaper for journeys shorter than 3 miles.

Answer to Question 20

a. Perpendicular gradients multiply to -1: $2 \times m = -1$, so $m = -\frac{1}{2}$.
b. Gradient $-\frac{1}{2}$, intercept 5: $y = -\frac{1}{2}x + 5$.
c. $2x + 1 = -0.5x + 5$, so $2.5x = 4$, $x = 1.6$ and $y = 2 \times 1.6 + 1 = 4.2$: $(1.6, 4.2)$.
d. The shortest distance is along the perpendicular: $\sqrt{1.6^2 + 0.8^2} = \sqrt{3.2} =$**1.8 units**.