Lesson - Speed, density and compound units

Year: Year 8

This lesson covers compound units, speed, average speed, density and unit pricing. The unit of a rate tells you which way to divide.

Starter activity

A car travels 60 miles in 1 hour. Write its speed in miles per hour. Now the car travels at the same speed for 3 hours. How far does it go? And how long does it take to go 150 miles? Write your findings as three formulae and say what each letter means.

What is a compound unit?

Definition: compound unit

A **compound unit** combines two different units, such as kilometres per hour (km/h), metres per second (m/s) or grams per cubic centimetre (g/cm^3). It describes a *rate*: how much of one quantity there is for each unit of another.

Speed, distance and time

Rule: speed, distance and time

$$\text{speed} = \frac{\text{distance}}{\text{time}}, \qquad \text{distance} = \text{speed} \times \text{time}, \qquad \text{time} = \frac{\text{distance}}{\text{speed}}$$

A formula triangle helps: put D at the top and S and T at the bottom. Cover the one you want and the triangle shows what to do.

Worked example 1: finding speed

A train travels 150 km in 2.5 hours. Find its speed.

Speed $= 150 \div 2.5 = 60$ km/h.

Worked example 2: distance and time

A car travels at 45 mph for 3 hours. A lorry travels 210 km at 70 km/h. Find the distance and the time.

Distance $= 45 \times 3 = 135$ miles. Time $= 210 \div 70 = 3$ hours.

Handling time carefully

Watch out: time is not decimal

There are 60 minutes in an hour, so 45 minutes is $\frac{45}{60} = 0.75$ hours and 1 hour 30 minutes is 1.5 hours. To convert a decimal back, multiply the decimal part by 60: 2.4 hours is 2 hours and $0.4 \times 60 = 24$ minutes, not 2 hours 40 minutes.

Worked example 3: minutes and hours

Find the speed for 12 km in 30 minutes, and the distance of a cyclist at 18 km/h for 20 minutes.

Change minutes to hours first. $12 \div 0.5 = 24$ km/h. For the cyclist, $18 \times \frac{1}{3} = 6$ km.

Changing units of speed

Method: km/h to m/s and back

1. To change km/h to m/s, multiply by 1000 to get metres.
2. Divide by 3600 to get seconds.
3. To go back from m/s to km/h, multiply by 3.6.

For example 72 km/h $= 72 \times 1000 \div 3600 = 20$ m/s and 15 m/s $= 54$ km/h.

Average speed

Rule: average speed

Average speed $= \frac{\text{total distance}}{\text{total time}}$. It is *not* the mean of the speeds.

Worked example 4: average speed

A driver travels 60 km at 40 km/h, then another 60 km at 60 km/h. Find the average speed.

First part: $60 \div 40 = 1.5$ h. Second part: $60 \div 60 = 1$ h. Total distance $= 120$ km, total time $= 2.5$ h. Average speed $= 120 \div 2.5 = 48$ km/h. The mean of 40 and 60 would give 50, which is wrong because more time is spent travelling slowly.

Density

Definition: density

Density tells you how much mass is packed into each unit of volume.

Rule: density, mass and volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}, \qquad \text{mass} = \text{density} \times \text{volume}, \qquad \text{volume} = \frac{\text{mass}}{\text{density}}$$

Worked example 5: density

Find the density of a block of mass 240 g and volume 30 cm^3, the mass of 50 cm^3 of aluminium (density 2.7 g/cm^3) and the volume of 156 g of iron (density 7.8 g/cm^3).

Density $= 240 \div 30 = 8$ g/cm^3. Mass $= 2.7 \times 50 = 135$ g. Volume $= 156 \div 7.8 = 20$ cm^3.

Unit pricing and population density

Key fact: other rates

Unit pricing is a rate: 2.5 kg for £4 is $4 \div 2.5 = £1.60$ per kg. To find the best value, compare the price for one unit of each. For population density, divide the number of people by the area: 6000 people in 4 km^2 is $6000 \div 4 = 1500$ people per km^2.

Worked example 6: matching units

A tank is 40 cm by 25 cm by 20 cm and full of water (density 1 g/cm^3). Find the mass of the water.

The volume is $40 \times 25 \times 20 = 20\,000$ cm^3. Mass $= 1 \times 20\,000 = 20\,000$ g $= 20$ kg. Check that units agree before you start: if the volume is in m^3 and the density in g/cm^3, convert one of them.

Note: why units matter

The unit of a rate tells you the calculation: kilometres per hour means kilometres divided by hours. If your answer has the wrong unit, the method is probably wrong. Always write the unit at the end, and check that the size makes sense: a walking speed of 400 km/h is clearly an error.

Watch out: common mistakes

- Using minutes with km/h without converting to hours.
- Treating 2.4 hours as 2 hours 40 minutes.
- Averaging two speeds instead of dividing total distance by total time.
- Turning the density formula upside down.
- Forgetting the units in the final answer.

Worked example 7: speed with a stop

A cyclist rides 30 km in 2 hours, stops for 30 minutes and then rides a further 10 km in 1 hour. Find her average speed for the whole trip and while she is moving.

Total distance $= 30 + 10 = 40$ km. Total time including the stop $= 2 + 0.5 + 1 = 3.5$ h, so average speed $= 40 \div 3.5 = 11.4$ km/h. Moving time $= 3$ h, so moving speed $= 40 \div 3 = 13.3$ km/h.

Worked example 8: changing units for density

A gold bar has dimensions 20 cm by 10 cm by 5 cm and a mass of 19.3 kg. Find its density in g/cm³ and in kg/m³.

Volume $= 20 \times 10 \times 5 = 1000$ cm³. Mass $= 19\,300$ g. Density $= 19300 \div 1000 = 19.3$ g/cm³. Since 1 g/cm³ is 1 000 kg/m³, the density is also $19\,300$ kg/m³.

Worked example 9: a rate problem

A tap fills a 90-litre barrel in 6 minutes. How long does it take to fill a 250-litre tank?

Rate $= 90 \div 6 = 15$ litres per minute. Time $= 250 \div 15 = 16.67$ minutes, which is 16 minutes 40 seconds. Using a rate turns any proportion problem into one division.

Worked example 10: m/s to km/h

The world record for 100 m is about 9.58 seconds. Find the speed in m/s and km/h.

Speed $= 100 \div 9.58 = 10.44$ m/s. Multiply by 3.6: $10.44 \times 3.6 = 37.6$ km/h. Check: in 1 hour (3 600 seconds) at 10.44 m/s the distance is $37\,584$ m $= 37.6$ km.

Note: the formula triangle

Put D at the top and S and T at the bottom of a triangle. Cover the quantity you want: covering D shows $S \times T$, covering S shows $D \div T$, covering T shows $D \div S$. The same triangle works for density with mass at the top, and for any rate. Before using it, check that your units match: km with hours, metres with seconds, grams with cm³.

Method: solving a compound unit problem

1. Write down what you know and what you need, with units.
2. Check that the units match, and convert minutes to hours or grams to kilograms if needed.
3. Write the formula, rearrange it, substitute and calculate.
4. Give the answer with the correct compound unit, and ask whether the size is sensible: a person does not run at 100 km/h.

Watch out: common mistakes

- Writing 2 h 30 min as 2.30 hours. It is 2.5 hours, because 30 min is half an hour.
- Averaging speeds: the average speed is total distance divided by total time.
- Mixing units: dividing km by minutes gives km/min, not km/h.
- Forgetting that 1 m³ is 1 000 litres, not 100.

Did you know?

The fastest land animal, the cheetah, can reach about 100 km/h, but only for a few hundred metres. In 1997 the car Thrust SSC became the first to break the sound barrier on land, at around 1 228 km/h in the Nevada desert, and the sound of its engines was louder than a jet at take-off.

The formula triangle and why units matter

Many pupils remember the three speed formulae with a triangle: distance at the top, speed and time at the bottom. Cover the quantity you want and the triangle shows the calculation: distance = speed × time, speed = distance ÷ time, time = distance ÷ speed. The same triangle works for density (mass at the top, density and volume at the bottom) and for many other rates. However, the triangle only gives the right answer when the units match. A speed in km/h must be used with a time in hours and a distance in kilometres; a density in g/cm³ needs a mass in grams and a volume in cm³. Converting first is the single most important step.

Example: mixing units

A cyclist rides at 24 km/h for 35 minutes. How far does she go?

The speed is in km per *hour*, so the time must be in hours: 35 minutes is $\frac{35}{60} = \frac{7}{12}$ of an hour. Distance = $24 \times \frac{7}{12} = 14$ km. Multiplying 24 by 35 would give 840 km, which is clearly impossible for a 35-minute ride: a quick sense check catches the mistake.

Did you know?

Ships and aircraft measure speed in knots: one knot is one nautical mile per hour, about 1.85 km/h. Sailors once measured speed by throwing a log tied to a knotted rope overboard and counting the knots that ran out in a set time.

Otto's tip

Look at the units in the question. km/h means "kilometres divided by hours", g/cm^3 means "grams divided by cubic centimetres". The unit tells you which way to divide.