Lesson - Solving linear equations

Year: Year 8

This lesson covers solving equations with inverse operations, brackets, unknowns on both sides, negative answers and forming equations from problems.

Starter activity

Look at these balance scales. On the left pan is a bag containing a secret number of counters, x, together with 3 extra counters. On the right pan are 11 counters. The scales are balanced. How many counters are in the bag? Now suppose the bag is on the left twice, with 3 counters: $2x + 3 = 11$. What do you do to both sides?

What is an equation?

Definition: equation and solving

An **equation** says that two expressions are equal, such as $3x - 4 = 11$. To **solve** it means to find the value of the unknown (here x) that makes the statement true.

Rule: the balance rule

Think of a pair of balanced scales: whatever you do to one side of an equation, you must do to the other, or the balance is lost.

Inverse operations

Definition: inverse operation

An **inverse operation** undoes another operation. Undo each operation using its inverse, working backwards from the unknown. The inverse of adding is subtracting, of multiplying is dividing, and of squaring is square-rooting.

Worked example 1: two-step equation

Solve $3x - 4 = 11$.

Add 4 to both sides: $3x = 15$. Divide both sides by 3: $x = 5$. Check: $3 \times 5 - 4 = 11$. Correct.

Worked example 2: fractions

Solve $\frac{x}{4} + 2 = 6$.

Subtract 2: $\frac{x}{4} = 4$. Multiply by 4: $x = 16$. When the unknown is divided by a number, multiplying both sides undoes it.

Equations with brackets

Method: equations with brackets

1. Expand the bracket, or divide both sides by the number outside if it divides everything exactly.
2. Collect the unknown terms on one side and the numbers on the other.
3. Solve, then check in the original equation.

Worked example 3: a bracket

Solve $2(x + 3) = 18$.

Expand to $2x + 6 = 18$, so $2x = 12$ and $x = 6$. Or divide both sides by 2 to get $x + 3 = 9$ straight away, so $x = 6$.

Unknowns on both sides

Key fact: collect the unknowns

Collect the unknowns on one side and the numbers on the other. A good habit is to remove the smaller x-term, so that you avoid negative coefficients.

Worked example 4: unknowns on both sides

Solve $5x + 3 = 2x + 18$.

Subtract $2x$: $3x + 3 = 18$. Subtract 3: $3x = 15$. Divide by 3: $x = 5$. Check: left $= 5 \times 5 + 3 = 28$, right $= 2 \times 5 + 18 = 28$.

Worked example 5: brackets on both sides

Solve $4(x - 1) = 2(x + 5)$.

Expand: $4x - 4 = 2x + 10$. Subtract $2x$: $2x - 4 = 10$. Add 4: $2x = 14$. Divide: $x = 7$. Check: left $= 4 \times 6 = 24$, right $= 2 \times 12 = 24$.

Negative answers and negative coefficients

Note: negative answers

Sometimes the answer is negative, and that is fine. Solve $2x + 3 = x - 4$: subtract x to

get $x + 3 = -4$, then subtract 3 to get $x = -7$. If the unknown has a minus sign, as in $12 - 3x = -6$, subtract 12 to get $-3x = -18$, then divide by -3 to get $x = 6$. Alternatively, add $3x$ to both sides to make the coefficient positive.

Forming equations from problems

Method: forming and solving an equation

1. Choose a letter for the unknown.
2. Write an equation using the information in the question.
3. Solve it.
4. Answer in the context of the question, and check.

Worked example 6: perimeter

A rectangle has length $(x + 5)$ cm and width x cm. Its perimeter is 38 cm. Find the dimensions.

Perimeter $= 2(x + 5) + 2x = 4x + 10$. So $4x + 10 = 38$, $4x = 28$, $x = 7$. The width is 7 cm and the length is 12 cm. Check: $2 \times 12 + 2 \times 7 = 38$.

Worked example 7: consecutive numbers

Three consecutive integers add up to 78. Find them.

Call them n, $n + 1$ and $n + 2$. Then $3n + 3 = 78$, so $3n = 75$ and $n = 25$. The numbers are 25, 26 and 27.

Worked example 8: comparing two deals

Company A charges £20 plus £5 per hour to hire a van. Company B charges £8 plus £8 per hour. For how many hours is the cost the same?

Let h be the number of hours. Then $20 + 5h = 8 + 8h$. Subtract $5h$: $20 = 8 + 3h$. Subtract 8: $12 = 3h$. So $h = 4$. Both charge $20 + 5 \times 4 = £40$. For a longer hire, company A is cheaper because its hourly rate is lower.

Which method should I use?

Note: choosing a method

Look at the shape of the equation first. If the unknown appears once, undo the operations in reverse order. If there are fractions, multiply every term by the common denominator. If the unknown is on both sides, collect the terms. Show each line of working, because method marks are given even when the final answer is wrong. Substitute your answer into the original equation to check it.

Watch out: common mistakes

- Doing an operation to only one side of the equation.
- Subtracting when you should add, for example $3x - 4 = 11$ becoming $3x = 7$.
- Not multiplying every term when you clear a fraction or expand a bracket.
- Making sign errors when subtracting a negative term.
- Giving the value of x when the question asks for something else, such as the perimeter.

Worked example 9: forming an equation from an angle problem

In a triangle the angles are x, $2x$ and $x + 40$ degrees. Find the angles.

The angles add to 180, so $x + 2x + x + 40 = 180$. Collect: $4x + 40 = 180$. Subtract 40: $4x = 140$. Divide by 4: $x = 35$. The angles are 35°, 70° and 75° (and $35 + 70 + 75 = 180$).

Worked example 10: an equation with fractions on both sides

Solve $\frac{x + 3}{2} = \frac{x - 1}{3} + 2$.

Multiply every term by 6 (the LCM of 2 and 3): $3(x + 3) = 2(x - 1) + 12$. Expand: $3x + 9 = 2x - 2 + 12$. Simplify: $3x + 9 = 2x + 10$. Subtract $2x$: $x + 9 = 10$, so $x = 1$. Check: left $= \frac{4}{2} = 2$, right $= 0 + 2 = 2$.

Worked example 11: choosing the unknown

Anna is 5 years older than Ben. The sum of their ages is 33. How old is each?

Let Ben's age be b. Anna's age is $b + 5$. Then $b + b + 5 = 33$, so $2b = 28$ and $b = 14$. Ben is 14 and Anna is 19. Always finish by answering the question in words.

Worked example 12: an equation with a negative coefficient

Solve $10 - 3x = 2x - 5$.

Add $3x$ to both sides to make the x term positive: $10 = 5x - 5$. Add 5: $15 = 5x$. Divide by 5: $x = 3$. Check: left $= 10 - 9 = 1$, right $= 6 - 5 = 1$.

Method: choose the best first step

1. Is there a fraction? Multiply every term by the common denominator.
2. Is there a bracket? Expand it.
3. Collect the x terms on one side and the numbers on the other.
4. Divide by the coefficient of x, then check by substituting.

Worked example 13: comparing two phone tariffs

Plan A costs £10 per month plus 5p per minute. Plan B costs £4 per month plus 11p per minute. For how many minutes do they cost the same, and which plan is better for 120 minutes?

Work in pence: $1000 + 5m = 400 + 11m$. Subtract $5m$: $1000 = 400 + 6m$. Subtract 400: $600 = 6m$, so $m = 100$ minutes. At 120 minutes, Plan A costs $1000 + 600 = 1600$p and Plan B costs $400 + 1320 = 1720$p, so Plan A is cheaper by £1.20. For fewer than 100 minutes, Plan B is cheaper.

Note: setting out your working

Write each line under the previous one, with the equals signs lined up. State what you do on the right-hand side of the line, for example (-4) or $(\div 3)$. This habit helps you to find slips and earns method marks even if your final answer is wrong. At the end, write the answer as a full sentence that uses the units of the question.

Watch out: balance errors

- Doing a step to only one side. If you add 4 to the left, you must add 4 to the right.
- Dropping a negative sign: $5 - 4x = 17$ gives $-4x = 12$, so $x = -3$, not 3.
- Forgetting to multiply the whole bracket: $3(x - 2) = 3x - 6$, not $3x - 2$.
- Finishing with $x =$ a number and not answering the question that was asked.

Did you know?

The equals sign was invented in 1557 by a Welsh mathematician named Robert Recorde, who wrote that no two things could be more equal than two parallel lines of the same length. Before that, mathematicians wrote the word *aequalis* in full. His book was the first English book about algebra.

Checking and explaining your solution

Every equation can be checked, and checking is the habit that separates a good answer from a lucky one. Substitute your value into the *original* equation, work out each side separately and confirm that they are equal. If they are not, look first at the signs when you expanded brackets, then at the step where you moved terms across. In a word problem, finish with a sentence that answers the question in context, such as “The family bought 3 child tickets”, and ask yourself whether the answer is sensible: a negative number of tickets or a fraction of a person tells you something has gone wrong.

Example: a check that finds an error

Kai solves $3(x - 2) = x + 8$ and gets $x = 5$. Is he right?

Left side: $3(5-2)=9$. Right side: $5+8=13$. The sides are different, so $x=5$ is wrong. Solving carefully: $3x-6=x+8$, so $2x=14$ and $x=7$. Check: $3\times5=15$ and $7+8=15$. Kai probably wrote $3x-2$ when expanding, forgetting to multiply the 2 by 3.

Otto's tip

Write each step on a new line and keep the equals signs lined up underneath each other. Neat working stops silly mistakes, and neat working shows the examiner your thinking.