Answers - Solving linear equations

Year: Year 8

Answer to Question 1

Undo the operations in reverse order with inverse operations, doing the same thing to both sides each time.

a. Subtract 9: $x = 8$. Check: $8 + 9 = 17$.
b. Divide by 5: $x = 9$.
c. Add 4: $3x = 15$. Divide by 3: $x = 5$. Check: $15 - 4 = 11$.
d. Subtract 2: $\frac{x}{4} = 4$. Multiply by 4: $x = 16$.
e. Subtract 7: $-2x = -6$. Divide by -2: $x = 3$. Check: $7 - 6 = 1$.

Answer to Question 2

a. Expand: $2x + 6 = 18$. Subtract 6: $2x = 12$. Divide by 2: $x = 6$. Check: $2 \times 9 = 18$.
b. Expand: $4x - 20 = 12$. Add 20: $4x = 32$. Divide by 4: $x = 8$. Check: $4 \times 3 = 12$.
c. Expand: $6x + 3 = 27$. Subtract 3: $6x = 24$, so $x = 4$. Check: $3 \times 9 = 27$.
d. Expand: $5x - 10 + 3 = 18$, so $5x - 7 = 18$. Add 7: $5x = 25$, so $x = 5$. Check: $5 \times 3 + 3 = 18$.

In (a) you could also divide both sides by 2 first: $x + 3 = 9$, giving the same answer more quickly.

Answer to Question 3

a. Subtract $2x$: $3x + 3 = 18$. Subtract 3: $3x = 15$, so $x = 5$. Check: $28 = 28$.
b. Subtract $3x$: $4x - 4 = 12$. Add 4: $4x = 16$, so $x = 4$. Check: $24 = 24$.
c. Subtract x: $3x + 9 = -6$. Subtract 9: $3x = -15$, so $x = -5$. Check: $-11 = -11$.
d. Expand: $2x + 8 = x + 11$. Subtract x: $x + 8 = 11$, so $x = 3$. Check: $14 = 14$.

Subtracting the smaller x term keeps the x coefficient positive, which avoids sign mistakes.

Answer to Question 4

a. Multiply both sides by 3: $x = 21$. Check: $21 \div 3 = 7$.
b. Multiply by 5: $x + 4 = 15$. Subtract 4: $x = 11$. Check: $15 \div 5 = 3$.
c. Add 1: $\frac{2x}{3} = 6$. Multiply by 3: $2x = 18$, so $x = 9$.
d. The LCM of 2 and 3 is 6. Multiply every term by 6: $3x + 2x = 60$, so $5x = 60$ and $x = 12$. Check: $6 + 4 = 10$.

Multiplying every term by the denominator removes the fractions in one step.

Answer to Question 5

a. Subtract x: $x + 3 = -4$. Subtract 3: $x = -7$. Check: $-11 = -11$.
b. Subtract 5: $3x = -3$. Divide by 3: $x = -1$. Check: $-3 + 5 = 2$.
c. Subtract 5: $-4x = 12$. Divide by -4: $x = -3$. Check: $5 + 12 = 17$.
d. Subtract $3x$: $5x + 1 = 4$. Subtract 1: $5x = 3$, so $x = \frac{3}{5}$. Check: both sides equal 5.8.

Negative and fractional answers are perfectly valid; do not change the equation to make the answer whole.

Answer to Question 6

a. $4n - 7 = 29$. Add 7: $4n = 36$. Divide by 4: $n = 9$.
b. $2(n + 5) = 24$. Divide by 2: $n + 5 = 12$, so $n = 7$.
c. $n + 12 = 3n$. Subtract n: $12 = 2n$, so $n = 6$. Check: $18 = 3 \times 6$.
d. $n + (n + 1) + (n + 2) = 51$, so $3n + 3 = 51$, $3n = 48$, $n = 16$. The numbers are **16, 17 and 18**.

For consecutive numbers, calling the smallest n makes the others $n + 1$ and $n + 2$, so only one letter is needed.

Answer to Question 7

a. Two adults cost $2 \times 5 = £10$ and c children cost $£3c$: $10 + 3c = 19$.
b. Subtract 10: $3c = 9$. Divide by 3: $c = 3$ children.
c. $3 \times 5 + 3c = 30$, so $15 + 3c = 30$, $3c = 15$, $c = 5$ children.
d. With a adults and a children: $5a + 3a = 56$, so $8a = 56$ and $a = 7$ adults (and 7 children).

Answer to Question 8

a. A: $20 + 5h$. B: $8 + 8h$.
b. $20 + 5h = 8 + 8h$, so $12 = 3h$, $h = 4$.
c. $20 + 5 \times 4 =$ **£40** (check B: $8 + 32 = 40$).
d. A: $20 + 30 = £50$. B: $8 + 48 = £56$. **Company A** saves her **£6**.

Company A has the higher fixed charge but the lower hourly rate, so it becomes the cheaper choice for any hire longer than 4 hours.

Answer to Question 9

a. £150 plus £4 for each of the p extra pupils: $150 + 4p = 210$.
b. Subtract 150: $4p = 60$. Divide by 4: $p = 15$.
c. p counts pupils over 20, so $20 + 15 =$**35 pupils** went.
d. Shared equally: $7n = 210$, so $n =$**30 pupils**.

Be careful to answer the question asked: p is only the number of pupils above 20, not the total.

Answer to Question 10

a. Perimeter = two lengths + two widths: $2(2x + 3) + 2x = 4x + 6 + 2x = 6x + 6 = 36$.
b. Subtract 6: $6x = 30$. Divide by 6: $x = 5$.
c. Width **5 m**, length $2 \times 5 + 3 =$**13 m**. Check: $2(13 + 5) = 36$.
d. Area = length × width: $13 \times 5 =$**65 m²**.
e. The fence goes round the 36 m perimeter: $36 \times 9.50 =$**£342**.

Answer to Question 11

a. $60 + 15m = 240$. Subtract 60: $15m = 180$. Divide by 15: $m = 12$ minutes.
b. $240 - 20m = 100$. Subtract 100 and add $20m$: $140 = 20m$, so $m = 7$ minutes.
c. $60 + 15m = 100 + 10m$. Subtract $10m$ and 60: $5m = 40$, so $m = 8$ minutes.
d. $60 + 15 \times 8 = 60 + 120 =$**180°C**; check the other oven: $100 + 80 = 180$.

Each part is a linear equation: write the temperature rule, set it equal to the target value, then solve.

Answer to Question 12

a. Use pounds: 8p is £0.08. $12 + 0.08t = 16.40$.
b. Subtract 12: $0.08t = 4.40$. Divide by 0.08: $t = 55$ extra texts.
c. $55 - 30 = 25$ extra texts: $12 + 0.08 \times 25 = 12 + 2 =$**£14**.
d. $12 + 0.08t = 10 + 0.12t$. Subtract 10 and $0.08t$: $2 = 0.04t$, so $t = 50$; each bill is $12 + 4 =$**£16**.

Answer to Question 13

a. Tom gains £6 a week: $35 + 6w$. Jess loses £3 a week: $80 - 3w$.
b. $35 + 6w = 80 - 3w$. Add $3w$ and subtract 35: $9w = 45$, so $w = 5$ weeks.
c. $35 + 6 \times 5 =$**£65** each (Jess: $80 - 15 = 65$).
d. $80 - 3w$ gives $3w > 60$, so $w > 20$. At 20 weeks she has exactly £20, so she first

has less in **week 21**.

Answer to Question 14

a. Weekly saving is $12 - 3 = 9$, so $45 + 9w$
b. $45 + 9w = 189$, $9w = 144$, $w = 16$ **weeks**
c. $75 + 9w = 189$, $9w = 114$, $w = 12.67$, so **13 weeks** (the money passes £189 during week 13).
d. No: after 15 weeks he has $45 + 135 = £180$, which is £9 short.

Each week adds £9, so the total after w weeks is the starting amount plus $9w$; rounding up in part (c) is needed because he must reach the full price.

Answer to Question 15

a. $3n + 8 = 5n - 6$. Subtract $3n$ and add 6: $14 = 2n$, so $n = 7$. Check: $29 = 29$.
b. $5(n + 4) = 3n + 30$. Expand: $5n + 20 = 3n + 30$. Then $2n = 10$, so $n = 5$. Check: $45 = 45$.
c. $n + \frac{n}{2} = 27$ means $\frac{3n}{2} = 27$. Multiply by 2 and divide by 3: $n = 18$.
d. **No**: the solution of Ali's puzzle is 7, which is positive.

Answer to Question 16

a. $4x - 12 = 2x + 6$, $2x = 18$, $x = 9$
b. Left: $4(9 - 3) = 24$. Right: $2 \times 9 + 6 = 24$. They match, so **Ali** is correct.
c. Bea: forgot the minus when expanding $(4x + 12)$. Cal: divided 18 by 6 instead of 2 (or wrong subtraction). Dev: expanded to $4x - 12$ but then subtracted the wrong terms to get $2x = 9$.

Substituting each wrong answer into the original equation would have shown at once that the two sides are not equal.

Answer to Question 17

a. $x + 2x + 10 + 3x - 10 = 180$, so $6x = 180$
b. $x = 30$
c. $30°$, $2 \times 30 + 10 = 70°$, $3 \times 30 - 10 = 80°$
d. Check $30 + 70 + 80 = 180$. All angles differ: **scalene**.
e. $4y + 20 = 180$, $y = 40$: angles $40°$**,** $40°$**,** $100°$ (isosceles).

The angle sum of 180° gives the equation; checking that the three angles really add up to 180° confirms the solution.

Answer to Question 18

a. Rectangle perimeter $2(x+4)+2\times 5$; square perimeter $4(x-1)$: $2(x+4)+10=4(x-1)$.
b. Expand: $2x+18=4x-4$. Subtract $2x$ and add 4: $22=2x$, so $x=11$.
c. Square side $11-1=$**10 cm**; rectangle **15 cm by 5 cm**. Both perimeters are 40 cm.
d. Square $10\times 10=100$ cm², rectangle $15\times 5=75$ cm². Equal perimeters do not mean equal areas: the **square** is larger.

Answer to Question 19

a. Every ticket that is not adult is a child ticket: $240-a$.
b. Income from adults $6a$ plus children $4(240-a)$: $6a+4(240-a)=1180$.
c. Expand: $6a+960-4a=1180$. Then $2a=220$, so $a=110$ adult tickets.
d. $240-110=$**130** child tickets. Check: $110\times 6+130\times 4=660+520=1180$.

Answer to Question 20

a. In 6 years both are 6 years older: Ben $x+6$, mother $4x+6$.
b. The mother will be three times Ben's age: $4x+6=3(x+6)$.
c. Expand: $4x+6=3x+18$. Subtract $3x$ and 6: $x=12$. Ben is 12.
d. Mother now $4\times 12=$**48**; in 6 years **54**, when Ben is 18, and $3\times 18=54$.