Lesson - Ratio and proportion

Year: Year 8

This lesson covers ratio, sharing in a ratio, direct and inverse proportion, best buys and scale factors.

Starter activity

To make lemonade, Dani mixes 3 glasses of lemon squash with 7 glasses of water. How many glasses of water does she need for 12 glasses of squash? What happens to the strength of the drink if she uses 3 squash and 6 water? Which of the two drinks is stronger? Explain with a ratio.

What is a ratio?

Definition: ratio

A **ratio** compares the sizes of two or more quantities. If a class has 12 boys and 18 girls, the ratio of boys to girls is $12 : 18$. The order matters, so "boys to girls" is not the same as "girls to boys".

Rule: simplifying a ratio

Like fractions, ratios can be simplified by dividing every part by the same number: $12 : 18 = 2 : 3$. Make sure the quantities are in the same units first: £1.50 and 50p become 150p and 50p, which gives $3 : 1$.

Note: the form 1 : n

Divide both parts by the first number to write a ratio as $1 : n$. For $4 : 10$, divide by 4 to get $1 : 2.5$. This form is useful for scales and comparing.

Sharing in a ratio

Method: sharing in a ratio

1. Add the parts of the ratio.
2. Divide the total by the number of parts to find the value of one part.
3. Multiply each part of the ratio by that value, and check that the shares add up to the total.

Worked example 1: sharing

Share £60 in the ratio $2:3$.

Parts $= 2 + 3 = 5$. One part $= 60 \div 5 = £12$. The shares are $2 \times 12 = £24$ and $3 \times 12 = £36$. Check: $24 + 36 = 60$.

Worked example 2: three parts

Share 90 in the ratio $2:3:4$.

Total parts $= 9$, so one part is $90 \div 9 = 10$ and the shares are 20, 30 and 40.

Using a ratio to find a missing amount

Key fact: find what one part is worth

If boys to girls is $3:5$ and there are 12 boys, then one part is $12 \div 3 = 4$ and there are $5 \times 4 = 20$ girls. If you are told the *difference* between two shares, that difference is some number of parts: for the ratio $2:3$ with a difference of 14, the difference is 1 part, so one part is 14 and the amounts are 28 and 42.

Direct proportion

Definition: direct proportion

Two quantities are in **direct proportion** when multiplying one by a number multiplies the other by the same number. On a graph, direct proportion gives a straight line through the origin, and the gradient is the cost (or amount) of one unit.

Method: the unitary method

1. Find the value of one item (divide).
2. Multiply by the number of items you want.

Worked example 3: unitary method

5 pens cost £3.50. Find the cost of 8 pens.

1 pen costs $3.50 \div 5 = 70$p. 8 pens cost $8 \times 70 = £5.60$. Similarly, if 4 people need 300 g of flour, 10 people need $300 \div 4 \times 10 = 750$ g.

Inverse proportion

Definition: inverse proportion

In **inverse proportion**, as one quantity goes up the other goes down so that the

product stays the same.

Worked example 4: workers and days

6 workers take 8 days to finish a job. How long do 4 workers take?

The total work is $6 \times 8 = 48$ worker-days. With 4 workers, the job takes $48 \div 4 = 12$ days.

Best buys

Worked example 5: best buy

A 500 g pack costs £1.80 and a 750 g pack costs £2.55. Which is better value?

Find the cost of the same amount, 100 g. The 500 g pack costs $180 \div 5 = 36$p per 100 g. The 750 g pack costs $255 \div 7.5 = 34$p per 100 g. The larger pack is the better value.

Scale factors, scale diagrams and maps

Rule: map scales

A map scale of $1 : 50\,000$ means that 1 cm on the map is 50 000 cm in real life, which is 500 m or 0.5 km. To go from the map to real life, multiply; to go the other way, divide.

Worked example 6: maps and models

A map has scale $1 : 25\,000$. How long on the map is a real distance of 3.5 km? A model has scale $1 : 20$ and is 15 cm high; what is the real height?

3.5 km $= 350\,000$ cm, and $350\,000 \div 25\,000 = 14$ cm. The model height represents $15 \times 20 = 300$ cm $= 3$ m.

Worked example 7: combining ratios

If $a : b = 2 : 3$ and $b : c = 4 : 5$, find $a : b : c$.

Make the number for b the same. The lowest common multiple of 3 and 4 is 12. Multiply the first ratio by 4: $8 : 12$. Multiply the second by 3: $12 : 15$. So $a : b : c = 8 : 12 : 15$.

Watch out: common mistakes

- Writing the ratio in the wrong order.
- Splitting £60 into 2 and 3 parts rather than 5 equal parts.

- Not converting to the same units before simplifying or using a scale.
- Using direct proportion for a situation which is inverse, such as workers and time.
- Comparing unit prices that are for different amounts.

Worked example 8: sharing with a difference

Aman and Beth share some sweets in the ratio $4:7$. Beth gets 18 more than Aman. How many sweets do they have in total?

The difference in parts is $7-4=3$ parts, and this is 18 sweets, so 1 part $=6$. Total parts $=11$, so the total is $11\times 6=66$ sweets. Aman gets 24 and Beth gets 42, and $42-24=18$.

Worked example 9: best buy using unit prices

A 750 g jar of jam costs £2.10 and a 1.2 kg jar costs £3.24. Which is the better buy?

Convert to the same unit: 1.2 kg $=1200$ g. Price per gram: $\frac{210}{750}=0.28$p and $\frac{324}{1200}=0.27$p. The larger jar is cheaper per gram, so the 1.2 kg jar is the better buy, by 0.01p per gram.

Worked example 10: a scale drawing

A scale drawing of a classroom is drawn at 1 cm to 2 m. The classroom is 4.5 cm by 3 cm on the drawing. Find its real size and floor area.

Real length $=4.5\times 2=9$ m. Real width $=3\times 2=6$ m. Area $=9\times 6=54$ m². Notice the area scale: 1 cm² on the drawing is $2\times 2=4$ m² in real life, and 13.5 cm² gives $13.5\times 4=54$ m².

Worked example 11: inverse proportion

It takes 8 taps 15 minutes to fill a pool. How long would 5 taps take?

The total work is $8\times 15=120$ tap-minutes. With 5 taps: $120\div 5=24$ minutes. Fewer taps take longer, so the answer must be bigger than 15 minutes: 24 is sensible.

Method: ratio problems with a bar model

1. Draw a bar divided into the number of parts in the ratio.
2. Label the given information (a total, a difference or one share) on the correct part of the bar.
3. Work out the value of one part.
4. Multiply to find each share, and check the shares add up to the total.

Watch out: common mistakes

- Dividing by the wrong number: in the ratio $2:3$ there are 5 parts, not 2 or 3.
- Not using the same units: change kilograms to grams before simplifying.
- Using direct proportion for an inverse problem: more workers means *less* time.
- Comparing prices without checking the amounts: always find the price per 100 g or per litre.

Did you know?

Ratio was used by the ancient Greeks to describe beautiful shapes. The Golden Ratio, about $1.618:1$, appears in the Parthenon in Athens and in many paintings, and the ratio of the sides of an A4 sheet of paper is $\sqrt{2}:1$, so that folding it in half gives another sheet with the same shape.

Direct or inverse?

Before you start a proportion question, decide which kind of proportion it is. Ask yourself: if one quantity doubles, what happens to the other? If the second quantity also doubles, as with the cost of more pens or the petrol for a longer journey, the quantities are in direct proportion and their *ratio* stays the same. If the second quantity halves, as with more workers finishing a job sooner, the quantities are in inverse proportion and their *product* stays the same. Choosing the wrong type gives answers that go the wrong way, so always check that your answer makes sense.

Example: deciding the type

It takes 3 pumps 8 hours to empty a tank. How long would 4 pumps take?

More pumps empty the tank faster, so this is inverse proportion. The job needs $3 \times 8 = 24$ pump-hours, and 4 pumps share them: $24 \div 4 = 6$ hours. If we had wrongly used direct proportion we would have found $8 \times \frac{4}{3} \approx 10.7$ hours, which is longer than before even though there are more pumps, a clear sign of an error.

Did you know?

Ordnance Survey Explorer maps, popular with walkers in the UK, use a scale of 1 : 25 000, so 4 cm on the map stands for exactly 1 km on the ground.

Otto's tip

Draw a bar model. Split a bar into equal parts, label each part with the value you find, and check that the parts add up to the total.