Test answers - Probability: outcomes and sample spaces

Year: Year 8 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. There are $4 + 6 + 10 = 20$ counters and 6 of them are blue. $P(\text{blue}) = \frac{6}{20}$, which simplifies to $\frac{3}{10}$ by dividing top and bottom by 2. (1 mark)

b. Ten counters are green, so $P(\text{green}) = \frac{10}{20}$. Then $P(\text{not green}) = 1 - \frac{10}{20} = \frac{1}{2}$. (1 mark)

c. Let n be the number of counters. Then $\frac{5}{n} = \frac{1}{4}$ (1 mark). Since 5 is a quarter of n, $n = 5 \times 4 =$**20** (1 mark).

Check (b) by counting: the 4 red and 6 blue counters are "not green", which is 10 out of 20. Check (c): $\frac{5}{20} = \frac{1}{4}$.

Answer to Question 2 (4 marks)

a. "Late" and "not late" are the only possibilities, so their probabilities add to 1: $1 - 0.15 =$**0.85**. (1 mark)

b. Expected number = probability × number of journeys: $0.15 \times 200 =$**30** late trains. (1 mark)

c. **No** (1 mark). The expected number in 20 journeys is $0.15 \times 20 = 3$, but this is only an average over many sets of journeys. In any particular 20 journeys the number of late trains could be 1, 2, 4 or another value because of chance, so "exactly 3" cannot be guaranteed (1 mark).

Over a much larger number of journeys the proportion of late trains will usually get close to 15%.

Answer to Question 3 (4 marks)

a. List each coin in order: **HH, HT, TH, TT**. HT and TH are different outcomes, because the first and second coins can be told apart. (1 mark)

b. Each of the 4 coin outcomes can be paired with any of the 6 die scores, giving $4 \times 6 =$**24** equally likely outcomes. (1 mark)

c. Two heads happens in 1 way (HH) and an even score in 3 ways (2, 4 or 6), so

there are $1 \times 3 = 3$ favourable outcomes (1 mark). The probability is $\frac{3}{24} = \frac{1}{8}$ (1 mark).

Check with independent events: $P(HH) = \frac{1}{4}$ and $P(\text{even}) = \frac{1}{2}$, and $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$.

Answer to Question 4 (4 marks)

a. A 6 by 6 sample space has 36 equally likely products. The pairs with product 12 are (2, 6), (6, 2), (3, 4) and (4, 3): 4 outcomes (1 mark). The probability is $\frac{4}{36} = \frac{1}{9}$ (1 mark).
b. A product is odd only when both scores are odd; one even score makes the product even. Each die has 3 odd scores (1, 3, 5), so there are $3 \times 3 = 9$ odd products (1 mark). The probability is $\frac{9}{36} = \frac{1}{4}$ (1 mark).

Drawing the full multiplication grid is the safest way to avoid missing pairs. The probability of an even product is therefore $1 - \frac{1}{4} = \frac{3}{4}$, which is why even products appear far more often.

Answer to Question 5 (4 marks)

a. Fill the Venn diagram from the middle: 5 play both. Football only is $18 - 5 = 13$ and tennis only is $12 - 5 = 7$ (1 mark for the regions). At least one sport: $13 + 5 + 7 = 25$, so neither $= 30 - 25 =$ **5** (1 mark).
b. Seven pupils play tennis only, out of 30 in the class: $\frac{7}{30}$. (1 mark)
c. Now only the 18 football players are considered, and 5 of them also play tennis: $\frac{5}{18}$. (1 mark)

Adding 18 and 12 to get 30 would count the 5 pupils who play both twice, which wrongly suggests nobody plays neither sport. In part (c) the denominator changes because we already know the pupil plays football.