Answers - Probability: outcomes and sample spaces

Year: Year 8

Answer to Question 1

An event that cannot happen has probability 0, an event that must happen has probability 1, and two equally likely outcomes each have probability one half.

a. **Impossible**, probability **0**.
b. **Even chance**, probability $\frac{1}{2}$.
c. 26 red cards out of 52: $\frac{1}{2}$, even chance.
d. **Certain** (or almost certain), probability **1**.
e. All of 1, 2, 3, 4, 5, 6 are less than 7: **certain**, probability **1**.

Answer to Question 2

a. There are 6 equally likely outcomes and one of them is 5: $\frac{1}{6}$.
b. The even numbers are 2, 4 and 6: $\frac{3}{6} = \frac{1}{2}$.
c. Greater than 4 means 5 or 6: $\frac{2}{6} = \frac{1}{3}$.
d. The primes are 2, 3 and 5 (1 is not prime): $\frac{3}{6} = \frac{1}{2}$.
e. P(3) $= \frac{1}{6}$, so P(not 3) $= 1 - \frac{1}{6} = \frac{5}{6}$.

Answer to Question 3

Each probability is the number of counters of that colour divided by the total number of counters.

a. $5 + 3 + 2 + 10 = \mathbf{20}$
b. $\frac{5}{20} = \frac{1}{4}$
c. $\frac{3+2}{20} = \frac{1}{4}$
d. $1 - \frac{10}{20} = \frac{1}{2}$
e. Let there be w white. $\frac{w}{20+w} = \frac{1}{5} \Rightarrow 5w = 20 + w \Rightarrow w = \mathbf{5}$.

Check (e): with 5 white counters there are 25 counters, and $\frac{5}{25} = \frac{1}{5}$.

Answer to Question 4

a. $\frac{1}{6} \times 120 = \mathbf{20}$
b. $0.35 \times 200 = \mathbf{70}$
c. $\frac{1}{8} \times 240 = \mathbf{30}$
d. Expected: $\frac{1}{6} \times 60 = 10$. Ali got 14, which is 4 more than expected. This is normal random variation.

The expected number is an average over many repetitions; in a single experiment the actual result usually differs a little from it.

Answer to Question 5

a. $\frac{27}{50} = \mathbf{0.54}$
b. $\frac{46}{100} = \mathbf{0.46}$
c. Priya's coin should be 0.5 and is 0.04 off. Sam's spinner should be 0.5 and is 0.04 off too: **both are equally close**.
d. More trials reduce the effect of chance, so the relative frequency settles closer to the true probability (the law of large numbers).

Answer to Question 6

Work systematically, for example by fixing the first choice and changing the second, so that no outcome is missed or repeated.

a. HH, HT, TH, TT: **4 outcomes**
b. HHH, HHT, HTH, HTT, THH, THT, TTH, TTT: **8 outcomes**
c. V-N, V-S, C-N, C-S, S-N, S-S: **6 outcomes**
d. $2 \times 2 = 4$, $2 \times 2 \times 2 = 8$, $3 \times 2 = 6$. Multiply the number of choices at each stage.

Answer to Question 7

a. Red $\frac{3}{8}$, blue $\frac{2}{8} = \frac{1}{4}$, green $\frac{1}{4}$, gold $\frac{1}{8}$.
b. $\frac{2+1}{8} = \frac{3}{8}$

c. Prizes: $\frac{1}{8} \times 200 = 25$. Sweets: $\frac{1}{4} \times 200 = 50$.
d. Income $= 200 \times 0.50 = £100$. Costs $= 25 \times 2 + 50 \times 0.05 = 50 + 2.50 = £52.50$. Profit $= 100 - 52.50 =$ **£47.50**.

The expected numbers of prizes and sweets come from probability × number of plays, so the profit is an estimate, not a guarantee.

Answer to Question 8

a. Main × drink gives 8 pairs: pasta-water, pasta-juice, pizza-water, pizza-juice, curry-water, curry-juice, salad-water, salad-juice.
b. $4 \times 2 \times 2 =$ **16 meals**
c. Pizza is chosen with probability $\frac{1}{4}$ because there are 4 mains and each is equally likely.
d. Only one meal is curry-juice-fruit out of 16: $\frac{1}{16}$
e. Without dessert: $4 \times 2 =$ **8 meals**

Multiplying the number of choices at each stage, $4 \times 2 \times 2$, counts every meal exactly once without listing them all.

Answer to Question 9

a. Relative frequency = scored ÷ taken. Ben $\frac{18}{24} =$**0.75**, Carl $\frac{21}{35} =$**0.6**, Dan $\frac{27}{45} =$ **0.6**.
b. **Ben** has the highest relative frequency, 0.75, even though Dan scored the most penalties.
c. Use Dan's relative frequency: $0.6 \times 20 =$**12** goals expected.
d. $\frac{4}{5} \times 35 = 28$ goals, which is $28 - 21 =$**7 more** than before.

Answer to Question 10

a. Rain or no rain covers everything: $1 - 0.35 =$**0.65**.
b. $1 - 0.6 =$**0.4**.
c. 0.6 is greater than 0.35, so rain is more likely on **Sunday**, by $0.6 - 0.35 =$**0.25**.
d. 80% of 150: $0.8 \times 150 =$**120** correct forecasts.
e. A probability of 0 means the forecasters think snow is **impossible** this weekend.

Answer to Question 11

a. $\frac{5}{400} = \frac{1}{80}$

b. All 5 of his tickets are still in the draw, so the probability is $\frac{5}{399}$.

c. $\frac{n}{400} = \frac{1}{10}$, so $n =$ **40 tickets**

d. Yes: $\frac{40}{400} = \frac{1}{10}$.

Each probability is the number of Ahmed's tickets divided by the number of tickets still in the draw, which is why the denominator falls from 400 to 399.

Answer to Question 12

a. A: $\frac{15}{500} = 0.03$. B: $\frac{28}{800} = 0.035$. C: $\frac{36}{1200} = 0.03$.

b. Total faulty $= 15 + 28 + 36 = 79$. Total tested $= 2500$. $\frac{79}{2500} =$ **0.0316**

c. $0.0316 \times 20000 =$ **632 bulbs**

d. Batch B (3.5%) is slightly higher than 3%, but the difference is small and could be due to chance. It is **not strong evidence**.

Combining all 2 500 bulbs gives a better estimate than any single batch, because more trials make the relative frequency more reliable.

Answer to Question 13

a. The 7 pupils in both clubs are inside both circles. Only chess: $24 - 7 =$**17**. Only drama: $18 - 7 =$**11**.

b. At least one club: $17 + 7 + 11 = 35$. Neither: $60 - 35 =$**25**.

c. $\frac{35}{60} = \frac{7}{12}$.

d. Exactly one club means only chess or only drama: $\frac{17 + 11}{60} = \frac{28}{60} = \frac{7}{15}$.

Answer to Question 14

a. $\frac{13}{52} = \frac{1}{4}$

b. $\frac{4}{52} = \frac{1}{13}$

c. 26 red cards plus the 2 black kings $= 28$: $\frac{28}{52} = \frac{7}{13}$

d. 12 picture cards, so $\frac{52-12}{52} = \frac{40}{52} = \frac{10}{13}$

e. $\frac{1}{4} \times 104 =$ **26 hearts**

In part (c) the red kings are already counted among the 26 red cards, so only the 2 black kings are added; counting them twice is a common mistake.

Answer to Question 15

Each cell of the grid is one of 36 equally likely outcomes. Count the cells for each event and put the count over 36.

a. $6 \times 6 =$ **36**

b. Total 7 occurs 6 times: $\frac{6}{36} = \frac{1}{6}$

c. Totals 2, 3, 4: $1 + 2 + 3 = 6$ outcomes: $\frac{6}{36} = \frac{1}{6}$

d. Totals 3, 6, 9, 12: $2 + 5 + 4 + 1 = 12$ outcomes: $\frac{12}{36} = \frac{1}{3}$

e. **7** is the most likely, with probability $\frac{1}{6}$.

Answer to Question 16

a. Totals 2 to 6: $1 + 2 + 3 + 4 + 5 = 15$ outcomes. $\frac{15}{36} = \frac{5}{12}$

b. Totals 8 to 12: $5 + 4 + 3 + 2 + 1 = 15$ outcomes. $\frac{15}{36} = \frac{5}{12}$

c. Yes, **the game is fair**: both players have the same probability.

d. It is already fair. If Rani scored on 7 as well, she would have $\frac{21}{36}$, which would favour her.

e. $\frac{5}{12} \times 72 =$ **30 points** each.

Answer to Question 17

a. $50 - 10 =$ **40**

b. $40 = 30 + 28 - n(C \cap T)$, so $n(C \cap T) = 58 - 40 =$ **18**

c. Both: 18. Only console: $30 - 18 = 12$. Only tablet: $28 - 18 = 10$. Neither: 10. Total $12 + 18 + 10 + 10 = 50$.

d. $\frac{12}{50} = \frac{6}{25}$

e. Of the 28 tablet owners, 18 own a console: $\frac{18}{28} = \frac{9}{14}$

In part (e) the sample is only the 28 tablet owners, so the denominator is 28, not 50.

Answer to Question 18

a. $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$, $B = \{3, 6, 9, 12, 15, 18\}$, $C = \{2, 3, 5, 7, 11, 13, 17, 19\}$.
b. $A \cap B = \{6, 12, 18\}$ (multiples of 6).
c. $n(A \cup B) = 10 + 6 - 3 =$ **13**
d. $\frac{3}{20}$ and $\frac{8}{20} = \frac{2}{5}$.
e. Yes: **2** is even and prime, so $A \cap C = \{2\}$.

The formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ subtracts the 3 multiples of 6 so that they are not counted twice.

Answer to Question 19

a. Products: row 1: 1, 2, 3, 4. Row 2: 2, 4, 6, 8. Row 3: 3, 6, 9, 12.
b. $3 \times 4 =$ **12**
c. Even products: 2, 4, 2, 4, 6, 8, 6, 12 = 8 outcomes. $\frac{8}{12} = \frac{2}{3}$
d. Greater than 6: 8, 9, 12 = 3 outcomes: $\frac{3}{12} = \frac{1}{4}$
e. Squares: 1, 4, 4, 9 = 4 outcomes: $\frac{4}{12} = \frac{1}{3}$

Answer to Question 20

a. Relative frequency: $\frac{130}{200} =$ **0.65**.
b. Heads and tails cover every outcome: $1 - 0.65 =$ **0.35**.
c. Two independent flips: multiply, $0.65 \times 0.65 =$ **0.4225**.
d. HT has probability $0.65 \times 0.35 = 0.2275$, and TH the same, so together $2 \times 0.2275 =$ **0.455**.
e. $0.4225 + 0.2275 + 0.2275 + 0.1225 = 1$. The four outcomes cover every possibility, so their probabilities must add up to 1.