Test answers - Powers, roots and standard form

Year: Year 8 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. The square root asks which number multiplied by itself gives 196. Since $14 \times 14 = 196$, $\sqrt{196}$ =**14** (1 mark). The cube root asks which number cubed gives 125: $5 \times 5 \times 5 = 125$, so $\sqrt[3]{125}$ =**5** (1 mark).

b. When multiplying powers of the same base, add the indices; when dividing, subtract them. So $3^8 \times 3^2 = 3^{10}$, then $3^{10} \div 3^4 = 3^6$. (1 mark)

c. A negative index means the reciprocal: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$, which is 0.125. (1 mark)

A common mistake in (c) is writing -8 or -6. A negative index never makes the answer negative.

Answer to Question 2 (4 marks)

a. Use a factor tree or repeated division: $90 = 2 \times 45 = 2 \times 3 \times 15 = 2 \times 3 \times 3 \times 5$ (1 mark). In index form: $2 \times 3^2 \times 5$ (1 mark).

b. In the same way, $84 = 2^2 \times 3 \times 7$. The HCF uses the primes common to both, each with the lower power: one 2 and one 3. HCF $= 2 \times 3$ =**6**. (1 mark)

c. The LCM uses every prime that appears, each with the higher power: $2^2 \times 3^2 \times 5 \times 7 = 4 \times 9 \times 35$ =**1 260**. (1 mark)

Check: HCF × LCM $= 6 \times 1260 = 7\,560$, and $90 \times 84 = 7\,560$. This product rule holds for any two numbers.

Answer to Question 3 (4 marks)

a. Move the decimal point so that the number is between 1 and 10: 0.000 072 becomes 7.2. The point moved 5 places to the right, so the power is negative: 7.2×10^{-5}. (1 mark)

b. Multiply the numbers and add the powers: $4 \times 3 = 12$ and $10^5 \times 10^4 = 10^9$. Since 12 is not between 1 and 10, write $12 = 1.2 \times 10$: 1.2×10^{10}. (1 mark)

c. Divide the numbers and subtract the powers: $9 \div 3 = 3$ and $10^{8-(-2)} = 10^{10}$: 3×10^{10}. (1 mark)

d. Powers differ, so change to ordinary numbers first: $6\,000 + 400 = 6\,400 = 6.4 \times 10^3$. (1 mark)

Answer to Question 4 (4 marks)

a. The two numbers have the same power of 10, so subtract the front numbers: $(2.3 - 1.5) \times 10^8 = 0.8 \times 10^8$ (1 mark). The front number must be between 1 and 10, so $0.8 \times 10^8 = 8 \times 10^7$ **km** (1 mark).

b. Time = distance ÷ speed
$= \dfrac{2.3 \times 10^8}{3 \times 10^5} = \dfrac{2.3}{3} \times 10^3 = 0.7666\ldots \times 10^3 = 766.66\ldots$ s (1 mark). To 3 significant figures this is **767 s** (1 mark).

767 seconds is about 12.8 minutes. In other words, the sunlight seen on Mars left the Sun almost 13 minutes earlier. Check (a): 80 000 000 km is 80 million km, and $230 - 150 = 80$ million.

Answer to Question 5 (4 marks)

a. Doubling once each hour for 6 hours multiplies by $2^6 = 64$ (1 mark). The number is $500 \times 64 = 32\,000$, which in standard form is 3.2×10^4 (1 mark).

b. We need $500 \times 2^h > 1\,000\,000$, so $2^h > 1\,000\,000 \div 500 = 2\,000$ (1 mark). Powers of 2: $2^{10} = 1\,024$, which is too small, and $2^{11} = 2\,048$, which is enough. The culture first exceeds one million after **11 hours** (1 mark).

Check: after 10 hours there are $500 \times 1024 = 512\,000$ bacteria, and after 11 hours $1\,024\,000$. Growth like this, multiplying by the same factor each hour, is called exponential growth.