Worksheet - Powers, roots and standard form

Year: Year 8 · 20

Show your working.

Question 1

Question 1 – Powers and roots *(Section A · Skills practice)*

Learn your squares, cubes and roots. Work these out without a calculator.

a. 2^5
b. 3^4
c. $\sqrt{169}$
d. $\sqrt[3]{64}$
e. 10^6

Write down the two whole numbers between which $\sqrt{75}$ lies, and estimate its value.

Question 2

Question 2 – The index laws *(Section A · Skills practice)*

Write each answer as a single power. Then, where possible, find its value.

a. $2^3 \times 2^4$
b. $5^9 \div 5^6$
c. $(3^2)^3$
d. $\dfrac{7^5 \times 7^2}{7^4}$
e. 4^0

Question 3

Question 3 – Negative and zero indices *(Section A · Skills practice)*

Work out the value of each, giving fractions where needed.

a. 2^{-3}
b. 10^{-2}
c. 5^{-1}
d. $\left(\dfrac{1}{2}\right)^{-2}$
e. $3^{-2} \times 3^4$

Question 4

Question 4 – Prime factors, HCF and LCM *(Section A · Skills practice)*

Use factor trees to write each number as a product of prime factors (in index form).

a. 60
b. 72
c. 180
d. Use your answers to find the highest common factor (HCF) of 60 and 72.
e. Find the lowest common multiple (LCM) of 60 and 72.

Question 5

Question 5 – Writing numbers in standard form *(Section A · Skills practice)*

Write each number in standard form.

a. 45 000
b. 7 200 000
c. 0.0006
d. 0.000 034 5
e. 500

Which of these is NOT in standard form? Explain: 0.8×10^5, 3.2×10^{-4}, 12×10^3.

Question 6

Question 6 – Calculating with standard form *(Section A · Skills practice)*

Work these out and give each answer in standard form.

a. $(3 \times 10^4) \times (2 \times 10^3)$
b. $(8 \times 10^7) \div (4 \times 10^2)$
c. $(6 \times 10^5) \times (5 \times 10^4)$
d. $4.5 \times 10^3 + 6 \times 10^2$

Question 7

Question 7 – The population of the UK *(Section B · Real-life practice)*

The population of the United Kingdom is about 67 000 000. The population of London is about 9 000 000.

a. Write each population in standard form.
b. Work out the number of people in the UK who do not live in London, in standard form.
c. What fraction of the UK population lives in London? Give a decimal to 2 decimal places.

d. The UK has an area of about 2.4×10^5 km². Work out the average number of people per square kilometre, to the nearest whole number.

Question 8

Question 8 – Distance to the Sun and the Moon *(Section B · Real-life practice)*

Space scientists use standard form all the time. The Sun is about 150 million km from Earth. The Moon is about 384 000 km from Earth. Light travels at 3×10^5 km per second.

Object	Distance from Earth
The Sun	150 000 000 km
The Moon	384 000 km

a. Write both distances in standard form.
b. How many times further away is the Sun than the Moon? Give your answer to the nearest whole number.
c. How long does light take to travel from the Sun to the Earth? Give your answer in seconds and in minutes.
d. How long does light take from the Moon to Earth, to 1 decimal place of a second?

Question 9

Question 9 – Doubling bacteria *(Section B · Real-life practice)*

A scientist grows bacteria in a dish. At 9 am there is 1 bacterium. The number doubles every 20 minutes.

Time	Number of bacteria
9:00	$2^0 = 1$
9:20	$2^1 = 2$
9:40	$2^2 = 4$
10:00	?

a. Complete the table for 10:00 and 10:20 using powers of 2.
b. How many bacteria are there at 12 noon? Write as a power of 2 and as a number.
c. How many hours will it take for the number to pass 1 million? (Use $2^{20} \approx 1\,048\,576$.)
d. Write your answer to part (b) in standard form.

Question 10

Question 10 – Computer memory *(Section B · Real-life practice)*

Computer memory is measured in bytes. 1 kilobyte (KB) is 2^{10} bytes, and 1 megabyte (MB) is 2^{10} kilobytes.

a. How many bytes are in 1 KB?
b. Write the number of bytes in 1 MB as a power of 2 and as a number.
c. A phone has 64 GB of storage, where 1 GB is 2^{10} MB. Write 64 GB in bytes as a power of 2.
d. A photo is about 2^{22} bytes. How many photos fit on the phone? Use the index laws.

Question 11

Question 11 – Paper folding *(Section B · Real-life practice)*

A sheet of paper is 0.1 mm thick. Priya folds it in half again and again. Each fold doubles the thickness.

Folds	Layers	Thickness
0	1	0.1 mm
1	2	0.2 mm
2	4	0.4 mm
3	?	?

a. Complete the row for 3 folds.
b. How many layers after 10 folds? Write as a power of 2.
c. Find the thickness after 10 folds in mm and in cm.
d. How many folds would be needed to make a stack taller than a 1.8 m door? (Use a calculator and work in mm.)

Question 12

Question 12 – Light years *(Section B · Real-life practice)*

A light year is the distance that light travels in one year, about 9.5×10^{12} km. The nearest star after the Sun, Proxima Centauri, is 4.25 light years away.

a. Work out the distance to Proxima Centauri in km, in standard form to 3 significant figures.
b. A spacecraft travels at 5×10^{4} km per hour. How many hours would it take to reach Proxima Centauri? Give your answer in standard form.
c. Convert your answer to years (365 days of 24 hours) and round to the nearest thousand years.
d. Explain why standard form is useful for these numbers.

Question 13

Question 13 – Size of tiny things *(Section B · Real-life practice)*

Biologists measure tiny things in metres. A human hair is about 0.00007 m wide. A bacterium is about 2×10^{-6} m long. A virus is about 1×10^{-7} m across.

a. Write the width of a human hair in standard form.
b. Put the three sizes in order, smallest first.
c. How many times wider than a virus is a human hair?
d. How many bacteria laid end to end would match the width of a hair?

Question 14

Question 14 – Chessboard grains *(Section B · Real-life practice)*

Legend says that a clever pupil asked the king for 1 grain of rice on the first square of a chessboard, 2 on the second, 4 on the third and so on, doubling each time.

a. How many grains are on the 6th square? Write as a power of 2.
b. How many grains are on the last (64th) square? Give your answer as a power of 2.
c. Use a calculator to write 2^{63} in standard form to 3 significant figures.
d. About 50 grains of rice weigh 1 g. Find the mass of the grains on the last square in tonnes, in standard form (1 tonne = 10^6 g).

Question 15

Question 15 – Comparing countries *(Section C · Problem solving)*

The table shows the population and area of four countries.

Country	Population	Area (km²)
UK	6.7×10^7	2.4×10^5
Iceland	3.8×10^5	1.0×10^5
Singapore	5.9×10^6	7.2×10^2
Canada	3.9×10^7	1.0×10^7

a. Work out the population density (people per km²) of each country to the nearest whole number.
b. Which country is the most crowded and which is the least crowded?
c. How many times more crowded is Singapore than the UK? Round to the nearest whole number.
d. If the UK population grows by 1.2% to the nearest thousand, write the new population in standard form.

Question 16

Question 16 – Prime factors and a school timetable *(Section C · Problem solving)*

Two school bells ring at the start of lessons. Bell A rings every 12 minutes and bell B rings every 18 minutes. They ring together at 9:00 am.

a. Write 12 and 18 as products of prime factors.
b. Find the LCM of 12 and 18.
c. At what time do the bells next ring together?
d. How many times between 9:00 am and 12:00 noon (including 9:00) do they ring together?
e. Find the HCF of 12 and 18 and explain what it tells you.

Question 17

Question 17 – Spotting the laws *(Section C · Problem solving)*

Four pupils each simplified the same expression, $\frac{2^6 \times 2^3}{2^4}$. Their answers are shown.

Pupil	Answer
Ali	2^5
Bea	2^9
Cal	1^5
Dev	4^5

a. Work out the correct answer as a power of 2 and as a number.
b. Which pupil is correct? Find the mistake in each of the other three answers.
c. Evaluate $2^6 \times 2^3$ and 2^4 separately and divide to check.
d. Simplify $\frac{5^8 \times 5^{-3}}{5^2}$.

Question 18

Question 18 – Mixed standard form problem *(Section C · Problem solving)*

A factory makes tiny screws. Each screw has a mass of 2.5×10^{-3} kg. The factory makes 4×10^6 screws per day.

a. Find the total mass of one day's screws, in kg and in standard form.
b. Write the mass in tonnes (1 tonne = 10^3 kg).
c. A lorry can carry 8×10^3 kg. How many lorry loads are needed for one day's screws?
d. Over 5 working days, what is the total mass in standard form?

Question 19

Question 19 – Cube numbers in a game *(Section C · Problem solving)*

A board game uses square and cube numbers. A player moves along a path of squares numbered 1 to 100. A token lands on every square number, and a bonus token lands on every cube number.

a. List the square numbers up to 100 and the cube numbers up to 100.
b. Which numbers between 1 and 100 are both square and cube? Use powers to explain.
c. How many squares have a token and how many have a bonus token?
d. A player lands on a square that is a cube and a square, and then moves forward $\sqrt{64}$ squares. Where does she end up if she started at 64?

Question 20

Question 20 – Challenge: standard form in estimating *(Section D · Challenge)*

Estimate how many seconds there are in a lifetime of 80 years, and then compare it with the number of heartbeats.

a. Work out the number of seconds in one year (365 days) and write it in standard form to 3 significant figures.
b. Find the number of seconds in 80 years, in standard form to 2 significant figures.
c. A heart beats about 70 times per minute. How many beats in 80 years? Write in standard form to 2 s.f.
d. If you spent $\frac{1}{3}$ of your life asleep, how many seconds is that? Give your answer in standard form to 2 s.f.