Answers - Powers, roots and standard form

Year: Year 8

Answer to Question 1

a. $2 \times 2 \times 2 \times 2 \times 2 = \mathbf{32}$
b. $3 \times 3 \times 3 \times 3 = \mathbf{81}$
c. $13^2 = 169$, so **13**
d. $4^3 = 64$, so **4**
e. **1 000 000**

$8^2 = 64$ and $9^2 = 81$, so $\sqrt{75}$ is between **8 and 9**. As 75 is closer to 81 than to 64, an estimate is about 8.7 (calculator: 8.66).

Knowing the squares up to 15^2 and the cubes up to 5^3 by heart makes roots like these instant.

Answer to Question 2

a. Multiplying powers of the same base: add the indices. $2^{3+4} = 2^7 =$**128**.
b. Dividing: subtract the indices. $5^{9-6} = 5^3 =$**125**.
c. A power of a power: multiply the indices. $3^{2\times3} = 3^6 =$**729**.
d. Top first: $7^{5+2} = 7^7$; then divide: $7^{7-4} = 7^3 =$**343**.
e. Any non-zero number to the power 0 is **1**, because $4^2 \div 4^2 = 4^0$ and also equals 1.

Answer to Question 3

a. A negative index means the reciprocal: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.
b. $10^{-2} = \frac{1}{10^2} = \frac{1}{100} =$**0.01**.
c. $5^{-1} = \frac{1}{5}$, the reciprocal of 5.
d. The reciprocal of $\frac{1}{2}$ is 2, so $\left(\frac{1}{2}\right)^{-2} = 2^2 =$**4**.
e. Add the indices: $3^{-2+4} = 3^2 =$**9**.

A negative index never makes the answer negative: it tells you to take the reciprocal of the positive power.

Answer to Question 4

Divide repeatedly by primes (2, 3, 5, ...) or draw a factor tree, then collect equal primes into powers.

a. $60 = 2^2 \times 3 \times 5$
b. $72 = 2^3 \times 3^2$
c. $180 = 2^2 \times 3^2 \times 5$
d. HCF: take the lowest powers of common primes: $2^2 \times 3 = \mathbf{12}$
e. LCM: take the highest powers: $2^3 \times 3^2 \times 5 = \mathbf{360}$

Check: $12 \times 360 = 4320 = 60 \times 72$; the product of the HCF and LCM equals the product of the two numbers.

Answer to Question 5

Standard form is $A \times 10^n$ with $1 \leq A$. Count how many places the decimal point moves: to the left for large numbers (positive n), to the right for small numbers (negative n).

a. 4.5×10^4
b. 7.2×10^6
c. 6×10^{-4}
d. 3.45×10^{-5}
e. 5×10^2

0.8×10^5 and 12×10^3 are not in standard form because the first number must be at least 1 and less than 10. The correct forms are 8×10^4 and 1.2×10^4.

Answer to Question 6

Multiply or divide the front numbers, use the index laws for the powers of 10, then adjust so that the front number is between 1 and 10.

a. $3 \times 2 = 6$ and $10^{4+3} = 10^7$: 6×10^7
b. $8 \div 4 = 2$ and $10^{7-2} = 10^5$: 2×10^5
c. $6 \times 5 = 30$, so $30 \times 10^9 = 3 \times 10^{10}$ (adjust to get a number between 1 and 10)
d. $4500 + 600 = 5100 = 5.1 \times 10^3$

Answer to Question 7

a. UK: 6.7×10^7. London: 9×10^6.
b. $6.7 \times 10^7 - 9 \times 10^6 = 6.7 \times 10^7 - 0.9 \times 10^7 = 5.8 \times 10^7$
c. $\dfrac{9 \times 10^6}{6.7 \times 10^7} = \mathbf{0.13}$

d. $\dfrac{6.7 \times 10^7}{2.4 \times 10^5} = 279.2$, so about **279 people per km²**

To subtract in standard form, first write both numbers with the same power of 10, as with $9 \times 10^6 = 0.9 \times 10^7$.

Answer to Question 8

a. Sun: 1.5×10^8 **km**. Moon: 3.84×10^5 **km**.
b. $\dfrac{1.5 \times 10^8}{3.84 \times 10^5} = 390.6$, so about **391 times**
c. $\dfrac{1.5 \times 10^8}{3 \times 10^5} = 5 \times 10^2 = 500$ seconds, which is $500 \div 60 =$ **8.3 minutes**
d. $\dfrac{3.84 \times 10^5}{3 \times 10^5} =$ **1.28 seconds**, about 1.3 s

Dividing numbers in standard form is quick: divide the front numbers and subtract the powers of 10.

Answer to Question 9

a. 10:00: $2^3 =$ **8**. 10:20: $2^4 =$ **16**.
b. From 9:00 to 12:00 is 180 minutes, which is 9 doublings. $2^9 =$ **512**.
c. 2^{20} needs 20 doublings, which is $20 \times 20 = 400$ minutes = **6 hours 40 minutes**.
d. $512 = 5.12 \times 10^2$

Each 20 minutes multiplies the number by 2, so after n doublings there are 2^n bacteria; three doublings happen every hour.

Answer to Question 10

a. 1 KB is $2^{10} =$**1 024 bytes**, not exactly 1 000.
b. 1 MB $= 2^{10}$ KB $= 2^{10} \times 2^{10} = 2^{20} =$**1 048 576 bytes**.
c. $64 = 2^6$, and each of GB, MB, KB brings another 2^{10}: $2^{6+10+10+10} = 2^{36}$ **bytes**.
d. Divide by subtracting indices: $2^{36} \div 2^{22} = 2^{14} =$**16 384 photos**.

Answer to Question 11

a. 3 folds: $2^3 =$ **8 layers**, thickness **0.8 mm**.
b. $2^{10} =$ **1024 layers**
c. $1024 \times 0.1 =$ **102.4 mm = 10.24 cm**
d. 1.8 m = 1800 mm, and $1800 \div 0.1 = 18000$ layers. $2^{14} = 16384$ is too small and

$2^{15} = 32768$ is enough, so **15 folds**.

The thickness after n folds is 0.1×2^n mm, so doubling quickly makes a sheet of paper enormously thick.

Answer to Question 12

a. $4.25 \times 9.5 \times 10^{12} = 40.375 \times 10^{12} = 4.04 \times 10^{13}$ **km**
b. $\dfrac{4.0375 \times 10^{13}}{5 \times 10^{4}} = 8.075 \times 10^{8}$ **hours**
c. $8.075 \times 10^{8} \div (365 \times 24) = 92180$ years, about **92 000 years**
d. The numbers are very large, so writing all the zeros would be slow and error-prone. Standard form shows the size of the number at a glance.

Dividing by $365 \times 24 = 8760$ hours in a year changes hours into years.

Answer to Question 13

a. Move the decimal point 5 places to the right to get 7: 7×10^{-5} **m**.
b. The more negative the power, the smaller the number: virus 1×10^{-7}, bacterium 2×10^{-6}, hair 7×10^{-5}. Order: **virus, bacterium, hair**.
c. $\dfrac{7 \times 10^{-5}}{1 \times 10^{-7}} = 7 \times 10^{2} =$**700 times**.
d. $\dfrac{7 \times 10^{-5}}{2 \times 10^{-6}} = 3.5 \times 10^{1} =$**35 bacteria**.

Dividing in standard form: divide the front numbers and subtract the powers of 10.

Answer to Question 14

a. The first square has 2^0, so the 6th has $2^5 =$ **32**
b. 2^{63}
c. $2^{63} = 9.22 \times 10^{18}$, so 9.22×10^{18}
d. $9.22 \times 10^{18} \div 50 = 1.844 \times 10^{17}$ g $= 1.84 \times 10^{11}$ **tonnes**

Square n holds 2^{n-1} grains because the first square starts at $2^0 = 1$; this is why square 64 holds 2^{63}.

Answer to Question 15

a. UK: $6.7 \times 10^{7} \div 2.4 \times 10^{5} \approx$ **279**. Iceland: $3.8 \times 10^{5} \div 10^{5} =$ **4**. Singapore: $5.9 \times 10^{6} \div 720 \approx$ **8194**. Canada: $3.9 \times 10^{7} \div 10^{7} =$ **4**.

b. **Singapore** is the most crowded. **Iceland** and **Canada** are the least crowded (about 4 per km²).
c. $8194 \div 279 \approx$ **29 times**
d. $6.7 \times 10^7 \times 1.012 = 6.7804 \times 10^7$, so 6.78×10^7 (to 3 s.f.).

Population density is population divided by area; the powers of 10 show quickly that Singapore's small area makes it far more crowded.

Answer to Question 16

a. $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$
b. LCM $= 2^2 \times 3^2 =$ **36**
c. **9:36 am**
d. Every 36 minutes. 3 hours = 180 minutes, and $180 \div 36 = 5$, so they ring together at 9:00, 9:36, 10:12, 10:48, 11:24 and 12:00: **6 times** including both ends (5 gaps).
e. HCF $= 2 \times 3 =$ **6**. The largest number of minutes that divides both 12 and 18 exactly, so the bells' timings can both be written as multiples of 6 minutes.

Answer to Question 17

a. $2^{6+3-4} = 2^5 =$ **32**
b. **Ali** is correct. Bea forgot to subtract 4. Cal changed the base to 1. Dev multiplied the base by the index law (turned 2 and 2 into 4).
c. $2^9 = 512$ and $2^4 = 16$, so $512 \div 16 = 32$. It matches.
d. $5^{8-3-2} = 5^3 =$ **125**

The index laws only work when the bases are the same, and the base never changes: only the indices are added or subtracted.

Answer to Question 18

a. $(2.5 \times 4) \times 10^{-3+6} = 10 \times 10^3 = 1 \times 10^4$ **kg**.
b. Divide by 1 000: $10^4 \div 10^3 =$ **10 tonnes**.
c. $\dfrac{1 \times 10^4}{8 \times 10^3} = 1.25$. One lorry is not enough, so round up to **2 lorry loads**.
d. $5 \times 1 \times 10^4 = 5 \times 10^4$ **kg**, which is 50 tonnes.

Multiplying in standard form: multiply the front numbers and add the powers of 10, then adjust the front number to lie between 1 and 10.

Answer to Question 19

a. Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. Cubes: 1, 8, 27, 64.
b. A number that is both is a sixth power: $1^6 = 1$ and $2^6 = 64$. So **1 and 64**.
c. **10** squares have tokens and **4** have bonus tokens (with 1 and 64 in both lists).
d. $\sqrt{64} = 8$, so she ends on square $64 + 8 =$ **72**.

Answer to Question 20

a. $365 \times 24 \times 60 \times 60 = 31\,536\,000$, which is 3.15×10^7 **s** to 3 s.f.
b. $80 \times 3.1536 \times 10^7 = 2.52 \times 10^9$, so 2.5×10^9 **s** to 2 s.f.
c. Minutes in 80 years: $60 \times 24 \times 365 \times 80 = 4.2 \times 10^7$; times 70 beats gives 2.94×10^9, so 2.9×10^9 **beats**.
d. $\dfrac{2.52 \times 10^9}{3} = 8.4 \times 10^8$ **s** asleep, about 27 years.

Rounding each answer to 2 significant figures is sensible because the 80 years and the 70 beats per minute are only estimates.