Lesson - Fractions, decimals and percentages

Year: Year 8

This lesson covers converting between fractions, decimals and percentages, ordering them, finding percentages of amounts, percentage change and reverse percentages.

Starter activity

A T-shirt costs £20. Shop A says "25% off". Shop B says "one quarter off". Shop C says "£5 off". Are all three shops offering the same price? Write 25% as a fraction and a decimal. Then find the price of a £60 jumper in each shop, and decide whether the shops are still equal.

Three ways to write the same number

Definition: percentage

A **percentage** means "parts per hundred", so 45% is 45 out of 100, which is the fraction $\frac{45}{100}$ and the decimal 0.45. Fractions, decimals and percentages are three ways of describing the same amount.

Rule: converting between the forms

- **Fraction to decimal:** divide the numerator by the denominator. $\frac{3}{8} = 3 \div 8 = 0.375$.
- **Decimal to percentage:** multiply by 100. $0.375 \times 100 = 37.5\%$.
- **Percentage to decimal:** divide by 100. $62\% = 0.62$.
- **Decimal to fraction:** read the place value, then simplify. $0.64 = \frac{64}{100} = \frac{16}{25}$.
- **Percentage to fraction:** put it over 100 and simplify. $45\% = \frac{45}{100} = \frac{9}{20}$.

Note: facts to learn by heart

$\frac{1}{2} = 0.5 = 50\%$, $\frac{1}{4} = 0.25 = 25\%$, $\frac{3}{4} = 0.75 = 75\%$, $\frac{1}{5} = 0.2 = 20\%$, $\frac{1}{10} = 0.1 = 10\%$, $\frac{1}{8} = 0.125 = 12.5\%$ and $\frac{1}{3} = 0.333\ldots \approx 33.3\%$.

Terminating and recurring decimals

Definition: terminating and recurring decimals

A **terminating decimal** ends, like 0.35. A **recurring decimal** repeats forever, like $0.333\ldots$. We write a recurring decimal with a dot above the repeating digit, or round it

sensibly.

Key fact: which fractions terminate

A fraction in its simplest form gives a terminating decimal when its denominator has no prime factors other than 2 and 5. So $\frac{7}{20} = 0.35$ terminates, but $\frac{1}{3}$ does not.

Comparing and ordering

Method: ordering fractions, decimals and percentages

1. Change everything to decimals (or all to percentages).
2. Line up the decimal points and add zeros if it helps.
3. Compare digit by digit, then write the original numbers in order.

Worked example 1: ordering

Put $\frac{5}{8}$, 0.6 and 59% in ascending order.

$\frac{5}{8} = 0.625$, 0.6 stays as it is and $59\% = 0.59$. Compare 0.625, 0.600 and 0.590. Ascending order: 59%, 0.6, $\frac{5}{8}$.

Fractions and percentages as operators

Rule: finding a fraction or percentage of an amount

To find $\frac{3}{5}$ of 85, divide by the denominator and multiply by the numerator: $85 \div 5 \times 3 = 51$. To find a percentage of an amount, use the decimal multiplier, or build it from 10% and 1%.

Worked example 2: percentage of an amount

Find 35% of 240.

With a multiplier: $0.35 \times 240 = 84$. By building up: 10% of 240 is 24, 5% is 12, so 35% is $24 + 24 + 24 + 12 = 84$.

One quantity as a percentage of another

Method: one quantity as a percentage of another

1. Make the units match.
2. Write the first quantity as a fraction of the second.
3. Multiply by 100.

Worked example 3: matching units

Write 45p as a percentage of £2.50.

Change £2.50 into 250p so both are in pence. Then $\frac{45}{250} \times 100 = 18\%$.

Percentage increase and decrease

Definition: multiplier

A **multiplier** is the number you multiply by to change an amount by a percentage. An increase of 15% leaves 115% of the original, so the multiplier is 1.15. A decrease of 24% leaves 76%, so the multiplier is 0.76.

Worked example 4: increase

Increase £80 by 15%.

The multiplier is 1.15, so $80 \times 1.15 = £92$.

Worked example 5: decrease

A coat costs £60 and is reduced by 35% in a sale. Find the sale price.

The multiplier is $1 - 0.35 = 0.65$, so the sale price is $60 \times 0.65 = £39$.

Percentage change

Rule: percentage change

$$\text{percentage change} = \frac{\text{actual change}}{\text{original}} \times 100\%$$

Always divide by the *original* amount, not the new one.

Worked example 6: train fare

A train fare goes up from £40 to £46. Find the percentage increase.

The change is £6, so $\frac{6}{40} \times 100 = 15\%$ increase.

Reverse percentages

Method: reverse percentages

1. Work out what percentage the final amount is of the original (for example, 120%

after a 20% increase).
2. Turn it into a multiplier.
3. Divide the final amount by the multiplier, then check by multiplying forwards.

Worked example 7: before the increase

A price after a 20% increase is £48. Find the original price.

£48 is 120% of the original. Divide by the multiplier: $48 \div 1.2 = £40$. Check: $40 \times 1.2 = 48$.

Watch out: common mistakes

- Dividing by the new value instead of the original when finding percentage change.
- Adding 15 instead of multiplying by 1.15, for example writing $80 + 15$.
- Forgetting to match units (pence and pounds) before making a fraction.
- Thinking a 20% rise followed by a 20% fall brings you back to the start. It does not: $1.2 \times 0.8 = 0.96$.
- Writing 5% as 0.5 instead of 0.05.

Note: choosing a method

For simple percentages such as 10%, 25% and 50%, mental methods are quick. For awkward percentages such as 17.5% or 8%, use the decimal multiplier on a calculator. To compare results such as test scores out of different totals, convert both to percentages first. At the end, check that your answer is sensible: a sale price should be smaller than the original price.

Worked example 8: percentage of a quantity in a table

In a class of 32 pupils, 12 own a dog, 9 own a cat and the rest have no pet. Find the percentage in each group.

Dogs: $\frac{12}{32} = 0.375 = 37.5\%$. Cats: $\frac{9}{32} = 0.28125 = 28.1\%$ (1 d.p.). No pet: $32 - 12 - 9 = 11$, and $\frac{11}{32} = 0.34375 = 34.4\%$ (1 d.p.). The unrounded percentages add to 100%. The rounded ones add to $37.5 + 28.1 + 34.4 = 100.0$.

Worked example 9: successive changes

A £200 bike is reduced by 10% and then by a further 10% in a clearance sale. What is the final price?

Do not add the percentages. First multiplier: 0.9. Second multiplier: 0.9. Combined multiplier: $0.9 \times 0.9 = 0.81$. Price $= 200 \times 0.81 = £162$. This is a 19% reduction, not

20%.

Worked example 10: the compound interest idea

£300 grows at 5% a year. Find its value after 3 years.

The multiplier for a 5% increase is 1.05. After 3 years you multiply three times: $300 \times 1.05^3 = 300 \times 1.157625 = £347.29$ (to the nearest penny). A simpler method is to use a table: year 1 £315, year 2 £330.75, year 3 £347.29.

Note: choosing the right percentage method

If the question gives an amount and a percentage and asks for a new amount, use a multiplier. If it gives two amounts and asks for a percentage, divide then multiply by 100. If it gives the amount *after* a change and asks for the original, divide by the multiplier. If you cannot decide, try the answer on a simple example such as £100. Always check that an increase gives a bigger answer and a decrease gives a smaller one.

Watch out: common mistakes

- Finding a percentage change and dividing by the *new* value instead of the *original* value.
- Adding percentages from different bases, for example "20% up then 20% down gives back the original".
- In a reverse percentage question multiplying by the percentage instead of dividing.
- Writing 0.05 as 50%: remember 5% is 0.05.

Did you know?

The word per cent comes from the Latin *per centum*, meaning "by the hundred". Roman emperors collected a tax of one hundredth on goods sold at auctions. Today, percentages are everywhere: nutrition labels, football statistics, bank interest and sale tags. The % sign evolved from an Italian manuscript from around 1425 where "per cento" was written as "p cento" and slowly turned into the symbol we use today. Estimating a percentage in your head, such as finding 10% by moving the decimal point one place, is a skill that shoppers, chefs and engineers use every single day.

Choosing the right method

Most percentage questions in Year 8 fall into four types, and recognising the type tells you the method. If you know the original amount and the percentage change, multiply by a single multiplier such as 1.15 or 0.88. If you know the original and the new amount, find the change and divide it by the *original* amount. If you know the new amount and the

percentage change, it is a reverse percentage, so divide by the multiplier. If several changes happen one after another, multiply all the multipliers together.

Example: which type is it?

After a 15% pay rise, Sam earns £460 a week. What did he earn before?

We know the new amount and the percentage, so this is a reverse percentage. £460 is 115% of the old wage, so the old wage is $460 \div 1.15 = £400$. Check: $400 \times 1.15 = 460$. Taking 15% off £460 instead would give £391, which is wrong because the 15% was calculated on the old wage, not on the new one.

Did you know?

Banks advertise savings rates as an AER, the annual equivalent rate, so that accounts paying interest monthly or yearly can be compared fairly using the same idea of multipliers you have met in this chapter.

Otto's tip

Always ask what 100% is. In a percentage change question 100% is the starting amount. Decide the multiplier first and the calculation becomes one line on your calculator.