Answers - Fractions, decimals and percentages

Year: Year 8

Answer to Question 1

To change a fraction to a decimal, divide the numerator by the denominator. Multiply a decimal by 100 to get a percentage, and write a percentage over 100 to get a fraction, then simplify.

a. $3 \div 8 = 0.375$, and $0.375 \times 100 =$ **37.5%**
b. $0.65 = \frac{65}{100} = \frac{13}{20}$, and as a percentage **65%**
c. $12\% = \frac{12}{100} = \frac{3}{25}$, as a decimal **0.12**
d. $7 \div 20 =$ **0.35**, which is **35%**

Answer to Question 2

a. $1 \div 8 =$ **0.125** (terminates)
b. $2 \div 9 = 0.\dot{2}$ (0.2222...)
c. $5 \div 11 = 0.\dot{4}\dot{5}$ (0.454545...)
d. $7 \div 15 = 0.4\dot{6}$ (0.4666...)

Only $\frac{1}{8}$ terminates. A fraction in its simplest form terminates only when the denominator has no prime factors other than 2 and 5. $8 = 2^3$ works, but $9 = 3^2$, 11 and $15 = 3 \times 5$ all contain other primes.

Answer to Question 3

Convert every number to a decimal, using the same number of decimal places, then compare.

a. As decimals: 0.6, 0.58, 0.62. Order: **0.58, $\frac{3}{5}$, 62%**
b. 0.875, 0.9, 0.85. Order: **85%, $\frac{7}{8}$, 0.9**
c. 0.3333..., 0.3, 0.335. Order: **0.3, $\frac{1}{3}$, 33.5%**
d. 0.45, 0.4, 0.46, 0.375. Order: **$\frac{3}{8}$, 0.4, $\frac{9}{20}$, 46%**

Fractions such as $\frac{1}{3}$ give recurring decimals, so write enough digits (0.3333...) to compare them accurately.

Answer to Question 4

a. 10% = 8, 5% = 4, so 15% = $8 + 4 =$ **£12**
b. 10% = 24, so 30% = 72 and 5% = 12, so 35% = $72 + 12 =$ **84 kg**
c. 10% = 38, so 5% = **19 m**
d. $0.175 \times 64 =$ **£11.20**

Building percentages from 10% and 5% avoids a calculator; the multiplier 0.175 is quicker when a calculator is allowed.

Answer to Question 5

a. $\frac{18}{24}$ simplifies to $\frac{3}{4}$, which is 0.75, so **75%**.
b. $\frac{7}{50} = \frac{14}{100}$, so **14%**.
c. Same units first: 2 m = 200 cm. $\frac{45}{200} = 0.225$, so **22.5%**.
d. 1.2 kg = 1 200 g. $\frac{250}{1200} = 0.2083\ldots$, so **20.8%** to 1 d.p.

In each case write a fraction with the part on top and the whole underneath, then multiply by 100.

Answer to Question 6

a. An increase of 15% means 115% of the amount, multiplier 1.15: $60 \times 1.15 =$**£69**.
b. A decrease of 12% leaves 88%, multiplier 0.88: $250 \times 0.88 =$**220 kg**.
c. 107.5% gives multiplier 1.075: $80 \times 1.075 =$**86 cm**.
d. 96% remains, multiplier 0.96: $1500 \times 0.96 =$**£1 440**.

A single multiplier does the change in one step: 100% plus the increase, or 100% minus the decrease, written as a decimal.

Answer to Question 7

a. Boots: $65 \times 0.8 = £52$. Tracksuit: $48 \times 0.85 = £40.80$. Bottle: $8 \times 0.95 = £7.60$. Rucksack: $36 \times 0.65 = £23.40$.
b. Reductions: boots £13, tracksuit £7.20, bottle £0.40, rucksack £12.60. **Boots** have the biggest reduction.
c. $52 + 40.80 =$ **£92.80**
d. $100 - 92.80 =$ **£7.20**
e. Original total $65 + 48 = 113$. Saving $= 113 - 92.80 = 20.20$. $\frac{20.20}{113} =$ **17.9%** to 1 d.p.

Answer to Question 8

a. Multiply each price by 1.2. Bag $45 \times 1.2 =$**£54**, keyboard $32.50 \times 1.2 =$**£39**, mouse $18 \times 1.2 =$**£21.60**.
b. Adding 20% gives 120% of the price, so the multiplier is **1.2**.
c. $54 + 39 + 21.60 =$**£114.60**.
d. Before VAT: $45 + 32.50 + 18 = £95.50$. VAT $= 114.60 - 95.50 =$**£19.10**, which is 20% of £95.50.

Answer to Question 9

a. Kai: $\frac{34}{40} = 85\%$. Lola: $\frac{27}{36} = 75\%$. Max: $\frac{41}{50} = 82\%$.
b. **Kai** did best (85%). **Lola** did worst (75%).
c. All three scored above 70%, so **all three passed**.
d. $27 \times 1.12 =$ **30.24**, so about **30.2 out of 36**.

Percentages allow a fair comparison of tests with different totals: 34 out of 40 is a better result than 41 out of 50, even though 41 is the larger mark.

Answer to Question 10

a. Increase $= 26950 - 24500 = 2450$. Divide by the original: $\frac{2450}{24500} = 0.1$, a **10% increase**.
b. Change $= 25130 - 26950 = -1820$. $\frac{1820}{26950} = 0.0675$, so a **6.8% decrease**.
c. $\frac{25130}{30000} = 0.8377$, so **83.8%** of the seats were filled.
d. Multiplier 1.12: $25130 \times 1.12 = 28145.6$, about **28 100 fans**, which fits in the 30 000 stadium.

Answer to Question 11

a. Plain: $0.6 \times 250 =$ **150 g**. Wholemeal: $250 - 150 =$ **100 g**.
b. $\frac{1}{3} = 0.3333 =$ **33.3%**
c. Flour: $250 \times 1.4 =$ **350 g**. Butter: $180 \times 1.4 =$ **252 g**.
d. Price per gram $= 1.70 \div 250 = 0.0068$. Cost $= 252 \times 0.0068 = 1.7136$, so **£1.71**.

Increasing by 40% means multiplying every ingredient by 1.4, so the recipe keeps the same proportions.

Answer to Question 12

a. $180000 \times 1.05 =$ **£189 000**
b. $189000 \times 0.97 =$ **£183 330**
c. Change $= 183330 - 180000 = 3330$. $\frac{3330}{180000} = 0.0185 =$ **1.9% increase**
d. No. The 3% is taken from the new, larger value £189 000, not from £180 000. The correct multiplier is $1.05 \times 0.97 = 1.0185$, so the overall increase is 1.85%, not 2%.

Answer to Question 13

a. $400 \times 1.03 =$ **£412**
b. $400 \times 1.03^2 =$ **£424.36**
c. $400 \times 1.03^3 = 437.09$. Interest $= 437.09 - 400 =$ **£37.09**
d. Simple interest: $400 + 3 \times 12 =$ **£436**. It is less because interest is paid only on the original £400, not on the interest already earned.

Compound interest grows faster because each year's interest is calculated on a larger balance.

Answer to Question 14

a. $90 \times 0.8 =$ **£72**
b. $72 \times 0.9 =$ **£64.80**
c. $\frac{90 - 64.80}{90} = 0.28 =$ **28%**
d. The multiplier is $0.8 \times 0.9 = 0.72$, which is 28% off, not 30%. The second 10% is taken from a smaller price.
e. 30% off gives $90 \times 0.7 = £63$. The difference is $64.80 - 63 =$ **£1.80** more.

Answer to Question 15

a. The sale price is 80% of the original: $48 \div 0.8 =$ **£60**
b. The VAT price is 120%: $18 \div 1.2 =$ **£15**
c. The new price is 105%: $63 \div 1.05 =$ **£60**
d. Dividing by 0.8 is correct. Multiplying by 1.2 would give £57.60, which would only be 20% of the *sale price* added on, but the 20% reduction was taken from the original price, which is bigger.

Answer to Question 16

a. Ashby: $+1800$, $\frac{1800}{12000} =$ **15% increase**. Barford: -850, $\frac{850}{8500} =$ **10% decrease**. Calton: $+1000$, $\frac{1000}{20000} =$ **5% increase**. Denby: $+1250$, $\frac{1250}{5000} =$ **25% increase**.
b. **Denby** (25%).
c. **Ashby** (an increase of 1800 people).
d. Multiplier 0.9: $7650 \times 0.9 = 6885$, so about **6890** people.

Percentage change compares the change with the starting size, which is why small Denby changed most in percentage terms while large Ashby gained the most people.

Answer to Question 17

a. TechWorld: $240 \times 0.75 = £180$. GameZone: $240 - 50 = £190$. ByteStore: $240 \times 0.9 \times 0.85 = £183.60$.
b. **TechWorld** at £180.
c. $0.9 \times 0.85 = 0.765$, so **23.5%** off.
d. Not in general. If the original price were £100 then 25% off gives £75 but £50 off gives £50. A fixed amount can beat a percentage depending on the price.

Answer to Question 18

a. Walk is already **45%**. Bus: $\frac{1}{4} =$**25%**. Car: $0.2 \times 100 =$**20%**.
b. The four groups make 100%: $100 - 45 - 25 - 20 =$**10%** cycle.
c. 10% of 240: $0.1 \times 240 =$**24 pupils**.
d. Each percentage of 360°: walk $0.45 \times 360 =$**162°**, bus **90°**, car **72°**, cycle **36°**. Check: $162 + 90 + 72 + 36 = 360$.

Answer to Question 19

a. $84 \times 0.9 =$ **£75.60**
b. $75.60 \times 0.125 =$ **£9.45**
c. Total $= 75.60 + 9.45 =$ **£85.05**. Each person pays $85.05 \div 6 =$ **£14.175**, so about **£14.18**.
d. $0.9 \times 1.125 =$ **1.0125**. Check: $84 \times 1.0125 = 85.05$.
e. $84 \times 1.125 = 94.50$; $94.50 \div 6 =$ **£15.75**.

Combining the multipliers 0.9×1.125 gives the whole calculation in one step; the voucher saves each person $15.75 - 14.18 = £1.57$.

Answer to Question 20

a. After December: $250 \times 1.2 = £300$. After January: $300 \times 0.8 = £240$.

b. No. It is £10 lower, because the 20% decrease is of the higher price £300.
c. $P \times 1.2 \times 0.8 = 0.96P$
d. 0.96 means a **4% decrease** overall.
e. Need to go from $1.2P$ back to P: multiplier $\frac{1}{1.2} = 0.8333$, so a decrease of **16.7%** (1 d.p.).