Lesson - Formulae and rearranging

Year: Year 8

This lesson covers algebra vocabulary, substituting into formulae and changing the subject of a formula. Always write the formula first, then the numbers, then the answer with units.

Starter activity

A taxi charges £3 to start the meter and then £2 for each mile. Write a rule in words for the price. Now replace the words by letters to make a formula. Use the formula to find the price of a 7-mile ride, and then work out how many miles you can travel for £21.

Words that matter in algebra

Definition: expression, equation, formula, identity

- An **expression** has no equals sign, for example $3x + 5$.
- An **equation** is true only for particular values, for example $3x + 5 = 17$.
- A **formula** links quantities using letters, for example $A = lw$ for the area of a rectangle.
- An **identity** is true for every value, for example $2(x + 1) = 2x + 2$.

Definition: inequality, term, factor

An **inequality** compares values using signs such as and $\geq$. A **term** is one part of an expression and a **factor** is a number or letter that divides exactly into it.

Substituting into formulae

Method: substituting

1. Write the formula.
2. Replace each letter with its value, putting negative values in brackets.
3. Follow the order of operations: brackets, indices, division and multiplication, addition and subtraction.
4. Write the answer with units.

Worked example 1: linear formula

Use $v = u + at$ to find v when $u = 3$, $a = 2$ and $t = 5$.

$v = 3 + 2 \times 5 = 3 + 10 = 13$. Multiplication comes before addition.

Worked example 2: temperature

The formula $F = \frac{9C}{5} + 32$ converts degrees Celsius to degrees Fahrenheit. Find F when $C = 20$.

$F = \frac{9 \times 20}{5} + 32 = 36 + 32 = 68$. So 20 degrees Celsius is 68 degrees Fahrenheit.

Worked example 3: negative numbers

Find y in $y = 3x - 2$ when $x = -4$, and find $2a^2$ when $a = 5$.

$y = 3 \times (-4) - 2 = -12 - 2 = -14$. For $2a^2$ the index applies only to a, so $2 \times 25 = 50$. Compare $(2a)^2 = 10^2 = 100$.

Changing the subject of a formula

Definition: subject of a formula

The **subject** of a formula is the letter on its own on one side, such as y in $y = 3x + 2$. To change the subject means to rearrange so that a different letter stands alone.

Rule: do the same to both sides

Use exactly the same inverse operations as when solving an equation. Do the same to both sides, and undo the operations in the reverse order from the way they were applied.

Method: changing the subject

1. List what was done to the new subject, in order. In $y = 2x + 7$, x was multiplied by 2 and then 7 was added.
2. Undo the steps backwards: subtract 7, then divide by 2.
3. Keep a whole side together over one fraction line.
4. Check by substituting numbers.

Worked example 4: one step

Make x the subject of $y = x + 5$, and s the subject of $P = 4s$.

Subtract 5 from both sides: $x = y - 5$. Divide both sides by 4: $s = \frac{P}{4}$.

Worked example 5: two steps

Make x the subject of $y = 2x + 7$.

Subtract 7: $y - 7 = 2x$. Divide both sides by 2: $x = \dfrac{y - 7}{2}$. The whole of $y - 7$ is divided by 2, so keep it together over one fraction line.

Worked example 6: a real formula

Make a the subject of $v = u + at$, then u.

Subtract u: $v - u = at$. Divide by t: $a = \dfrac{v - u}{t}$. To make u the subject, subtract at from both sides instead: $u = v - at$.

Worked example 7: area of a triangle

Make h the subject of $A = \dfrac{1}{2}bh$, then find h when $A = 24$ and $b = 8$.

Multiply both sides by 2: $2A = bh$. Divide by b: $h = \dfrac{2A}{b}$. With the numbers, $h = \dfrac{48}{8} = 6$.

Using rearranged formulae

Key fact: rearrange first or substitute first

You can either substitute first and then solve, or rearrange first and then substitute. For a formula you use many times, rearranging first saves effort.

Worked example 8: hiring a room

The cost of hiring a room for n people is $T = 5n + 20$. How many people can attend when the total is £95?

Rearrange: $n = \dfrac{T - 20}{5}$. Then $n = \dfrac{95 - 20}{5} = \dfrac{75}{5} = 15$ people.

Watch out: common mistakes

- Dividing only one term instead of the whole side, for example $x = y - \dfrac{7}{2}$.
- Subtracting when you should add; check the inverse of each operation.
- Forgetting brackets when substituting negative numbers.
- Doing addition before multiplication in a substitution.
- Leaving the new subject on both sides of the equals sign.

Worked example 9: rearranging a formula with a fraction

Make a the subject of $v = \dfrac{a+b}{2}$.

Multiply both sides by 2: $2v = a + b$. Subtract b from both sides: $2v - b = a$. So $a = 2v - b$. Check with numbers: if $a = 6$ and $b = 4$ then $v = 5$, and $2 \times 5 - 4 = 6$. Correct.

Worked example 10: a formula in context

A plumber charges $C = 35 + 22h$ pounds for a job of h hours. Find the cost of a 3.5-hour job and the time for a bill of £123.

For 3.5 hours: $C = 35 + 22 \times 3.5 = 35 + 77 = £112$. For £123: $35 + 22h = 123$, so $22h = 88$ and $h = 4$ hours. You can also rearrange first: $h = \dfrac{C - 35}{22} = \dfrac{88}{22} = 4$.

Worked example 11: substituting a negative number

Find y when $y = 2x^2 - 3x + 1$ and $x = -2$.

Put the negative number in brackets: $y = 2 \times (-2)^2 - 3 \times (-2) + 1$. Square first: $(-2)^2 = 4$, so $2 \times 4 = 8$. Then $-3 \times (-2) = +6$. So $y = 8 + 6 + 1 = 15$. If you forget the brackets, your calculator may square only the 2 and give a wrong answer.

Worked example 12: rearranging a squared term

Make x the subject of $y = \dfrac{x^2}{5} + 3$.

Subtract 3: $y - 3 = \dfrac{x^2}{5}$. Multiply by 5: $5(y - 3) = x^2$. Take the square root: $x = \sqrt{5(y-3)}$. For example, if $y = 8$ then $x = \sqrt{25} = 5$ (or -5, because $(-5)^2 = 25$ as well).

Worked example 13: choosing whether to rearrange first

The formula $T = 5n + 30$ gives the cooking time in minutes for a chicken of n kilograms. A chicken takes 80 minutes. Find its mass.

Method 1: solve the equation $5n + 30 = 80$, so $5n = 50$ and $n = 10$ kg. Method 2: rearrange first to get $n = \dfrac{T - 30}{5}$, then substitute $T = 80$ to get $n = \dfrac{50}{5} = 10$. Method 2 is better if you must answer many questions with different times, because you only rearrange once.

Worked example 14: a formula with two letters to find

The formula $s = \dfrac{d}{t}$ gives speed. Make d the subject, then find how far a cyclist rides in 2.5 hours at 14 miles per hour.

Multiply both sides by t: $d = st$. Substitute: $d = 14 \times 2.5 = 35$ miles. To find the time for 49 miles, make t the subject: $t = \dfrac{d}{s} = \dfrac{49}{14} = 3.5$ hours, which is 3 hours 30 minutes. Always check that your answer has sensible units and size: a cyclist cannot ride 3500 miles in an afternoon.

Method: changing the subject

1. Identify the letter you want and the operations done to it, in order.
2. Write down the inverse of each operation.
3. Apply the inverses to both sides, starting with the operation that was done last.
4. Check by substituting a pair of numbers into the original and the new formula.

Note: the same steps in the same order

When you change the subject, you use the same inverse operations as when you solve an equation. The only difference is that letters stay as letters. List the operations that have been done to the subject, then undo them in reverse order. Each step must be done to *both* sides. Write one step per line so that you or your teacher can check each line quickly.

Watch out: common mistakes

- Dividing only one term: from $y = 3x + 2$ the answer is $x = \dfrac{y - 2}{3}$, not $x = y - 2 \div 3$.
- Confusing an equation with a formula: an equation is solved to find a number, but a formula is rearranged to find a letter.
- Forgetting brackets when substituting a negative: $(-2)^2 = 4$ but $-2^2 = -4$.

Did you know?

The formula for converting temperatures was published in 1724 by Daniel Fahrenheit, who made the first reliable mercury thermometer. Scientists today use formulae every day: the formula $E = mc^2$ links energy and mass, and engineers use the formulae for area and speed whenever they design a bridge, a lift or a train timetable.

Rearranging with a flow diagram

A reliable way to change the subject of a formula is to draw what happens to the letter you want, step by step, in a flow diagram. For $y = \frac{3x - 4}{5}$, x is first multiplied by 3, then 4 is subtracted, then the result is divided by 5. To make x the subject, start from y and undo each step in reverse order with the inverse operation: multiply by 5, add 4, divide by 3. This gives $x = \frac{5y + 4}{3}$. The flow diagram makes the order of the inverse operations clear and stops you from undoing them in the wrong order.

Example: checking a rearrangement

Check that $x = \frac{5y + 4}{3}$ is a correct rearrangement of $y = \frac{3x - 4}{5}$.

Choose a value for x, for example $x = 8$. The original formula gives $y = \frac{24 - 4}{5} = 4$. Now put $y = 4$ into the new formula: $x = \frac{20 + 4}{3} = 8$. We get back to the value we started with, so the rearrangement is correct. This check takes less than a minute and catches almost every sign error.

Did you know?

Spreadsheets and calculators work with formulae all the time. In a spreadsheet, a cell containing =3.2+1.8*B2 is a formula with B2 playing the part of the letter m, and changing the number in B2 updates the answer instantly.

Otto's tip

Put numbers into your rearranged formula to check it. If the original formula gives $y = 13$ when $x = 3$, your new formula should give $x = 3$ when you put in $y = 13$.