Answers - Formulae and rearranging

Year: Year 8

Answer to Question 1

a. **Expression**: no equals sign, it just groups terms.
b. **Equation**: it can be solved, $x = 5$.
c. **Formula**: it connects quantities with letters (area, length, width) in a real-life rule.
d. **Identity**: it is true for every value of x, shown by the sign $\equiv$.
e. **Inequality**: it uses $>$, so the solution is a range of values, $y > 3$.

The key differences: an expression has no equals sign, an equation is true only for particular values, a formula links real quantities, and an identity is true for every value.

Answer to Question 2

a. Substitute $l = 8$, $w = 5$: $P = 2 \times 8 + 2 \times 5 = 16 + 10 =$**26**.
b. Multiplication first: $v = 4 + 3 \times 6 = 4 + 18 =$**22**.
c. $C = 5 \times 12 + 20 = 60 + 20 =$**80**.
d. $A = \frac{1}{2} \times 9 \times 14 = \frac{1}{2} \times 126 =$**63**.

In every case write the formula with the numbers in place before calculating, then follow the order of operations.

Writing each substituted formula on a separate line makes your method easy to follow and earns method marks even if the arithmetic slips.

Answer to Question 3

a. $ut = 5 \times 3 = 15$ and $\frac{1}{2}at^2 = \frac{1}{2} \times 4 \times 9 = 18$. So $s = 15 + 18 =$**33**.
b. $(-2)^2 = 4$, so $3 \times 4 = 12$; and $-2 \times (-2) = +4$. $y = 12 + 4 =$**16**.
c. Square c first: $10^2 = 100$, then $E = 2 \times 100 =$**200**.
d. $u^2 = 100$ and $2as = 2 \times (-3) \times 4 = -24$. $V = 100 - (-24) =$**124**.

Brackets round negative values and squaring before multiplying are the two things that matter most here.

Answer to Question 4

a. x has 9 added, so subtract 9 from both sides: $x = y - 9$.
b. x has 4 subtracted, so add 4 to both sides: $x = y + 4$.

c. x is multiplied by 5, so divide both sides by 5: $x = \frac{y}{5}$.
d. x is divided by 3, so multiply both sides by 3: $x = 3y$.

Check (c) with $x = 2$: $y = 10$ and $\frac{10}{5} = 2$.

Answer to Question 5

Undo the operations in reverse order: first remove the term that is added or subtracted, then divide by the number multiplying the new subject.

a. $y - 2 = 3x$, so $x = \frac{y - 2}{3}$
b. $P - 2w = 2l$, so $l = \frac{P - 2w}{2}$
c. $C - 20 = 5n$, so $n = \frac{C - 20}{5}$
d. $F - 32 = \frac{9C}{5}$. Multiply by 5: $5(F - 32) = 9C$. Divide by 9: $C = \frac{5(F - 32)}{9}$

Check (a) with $x = 4$: $y = 14$, and $\frac{14 - 2}{3} = 4$.

Answer to Question 6

When the subject is squared, rearrange to get r^2 on its own, then take the square root. When it is under a square root, square both sides.

a. $r^2 = \frac{A}{\pi}$, so $r = \sqrt{\frac{A}{\pi}} = \sqrt{50 \div \pi} =$ **4.0**
b. $r^2 = \frac{V}{\pi h}$, so $r = \sqrt{\frac{V}{\pi h}} = \sqrt{300 \div (6\pi)} =$ **4.0**
c. Square both sides: $r = s^2$. $r = 7^2 =$ **49**
d. $4d = r^2$, so $r = \sqrt{4d} = \sqrt{36} =$ **6**

Answer to Question 7

a. $F = 3.2 + 1.8 \times 5 = 3.2 + 9 =$ **£12.20**
b. $F = 3.2 + 1.8 \times 12.5 = 3.2 + 22.5 =$ **£25.70**
c. $F - 3.2 = 1.8m$, so $m = \frac{F - 3.2}{1.8}$
d. $m = \frac{20 - 3.2}{1.8} = \frac{16.8}{1.8} =$ **9.3 miles** (9.3 miles)
e. Metered fare for 14 miles: $3.2 + 1.8 \times 14 = 3.2 + 25.2 = £28.40$. The fixed price of £25 is **cheaper by £3.40**.

In part (d) the answer is rounded down to 9.3 miles, because 9.4 miles would cost slightly more than £20.

Answer to Question 8

a. Edinburgh: $1.8 \times 12 + 32 =$ **53.6°F**. Madrid: $1.8 \times 35 + 32 =$ **95°F**. Reykjavik: $1.8 \times (-5) + 32 =$ **23°F**.
b. $F - 32 = 1.8C$, so $C = \dfrac{F - 32}{1.8}$
c. $C = \dfrac{77 - 32}{1.8} = \dfrac{45}{1.8} =$ **25°C**
d. $2C = 1.8C + 32 \Rightarrow 0.2C = 32 \Rightarrow C =$ **160°C** (and 320°F).

Check part (c) with the original formula: $1.8 \times 25 + 32 = 45 + 32 = 77$.

Part (d) has exactly one answer: below 160°C the Fahrenheit value is more than double the Celsius value, and above 160°C it is less.

Answer to Question 9

a. $C = 45 + 12 \times 4 = 45 + 48 =$**£93**.
b. Subtract 45: $C - 45 = 12h$. Divide by 12: $h = \dfrac{C - 45}{12}$.
c. $h = \dfrac{150 - 45}{12} = \dfrac{105}{12} = 8.75$. Hiring is by the hour, so she can afford **8 hours** (9 hours would cost £153).
d. $45 + 12h = 20h + 15$. Subtract $12h$ and 15: $30 = 8h$, so $h = 3.75$ hours, when both cost £90.

Answer to Question 10

a. $d = st = 80 \times 2.5 =$**200 miles**.
b. Divide both sides by t: $s = \dfrac{d}{t}$.
c. Divide both sides by s: $t = \dfrac{d}{s}$.
d. $t = \dfrac{190}{95} =$**2 hours** exactly (2 hours 0 minutes).
e. $s = \dfrac{270}{3} =$**90 mph**.

The three forms $d = st$, $s = \dfrac{d}{t}$ and $t = \dfrac{d}{s}$ are the same relationship; choose the one whose subject is the quantity you need.

Answer to Question 11

Substitute the units and the days into the formula. To rearrange, subtract the standing charge $0.60d$ first, then divide by the price per unit 0.28.

a. March: $0.28 \times 150 + 0.60 \times 31 = 42 + 18.60 =$ **£60.60**. June: $0.28 \times 90 + 0.60 \times 30 = 25.20 + 18 =$ **£43.20**.

b. $B - 0.60d = 0.28u$, so $u = \dfrac{B - 0.60d}{0.28}$

c. $u = \dfrac{62.40 - 18}{0.28} = \dfrac{44.40}{0.28} =$ **159 units** (about 159)

d. 200 units: $56 + 18 = £74$. 100 units: $28 + 18 = £46$, twice that is £92. £74 is **not** more than £92, because the standing charge is paid once.

Answer to Question 12

a. Subtract $2l$: $P - 2l = 2w$. Divide by 2: $w = \dfrac{P - 2l}{2}$.

b. $w = \dfrac{360 - 2 \times 120}{2} = \dfrac{120}{2} =$**60 m**.

c. The fence goes round the perimeter: $360 \times 14.50 =$**£5 220**.

d. Length $l = 2w$: $P = 2(2w) + 2w = 6w = 300$, so $w =$**50 m** and $l =$**100 m**.

Check part (b): $2 \times 120 + 2 \times 60 = 240 + 120 = 360$ m, the given perimeter.

Answer to Question 13

a. $P = 12 + 2.5 \times 16 = 12 + 40 =$**£52**.

b. Subtract 12, then divide by 2.5: $n = \dfrac{P - 12}{2.5}$.

c. $n = \dfrac{50 - 12}{2.5} = 15.2$. He cannot buy part of a portion, so **15 portions** (£49.50).

d. The £8 is added to the fixed part: $P = 20 + 2.5n$. For 20 portions: $20 + 50 =$**£70**.

Each extra portion adds £2.50, while the £12 (or £20) is paid once, whatever the size of the cake.

Answer to Question 14

a. $I = \dfrac{800 \times 3 \times 4}{100} = \dfrac{9600}{100} =$**£96**.

b. Multiply by 100: $100I = PRT$. Divide by RT: $P = \dfrac{100I}{RT}$.

c. $P = \dfrac{100 \times 45}{2.5 \times 3} = \dfrac{4500}{7.5} =$**£600**.

d. In the same way $T = \frac{100I}{PR}$: $T = \frac{10000}{2000}$ =**5 years**.

Check (c): $\frac{600 \times 2.5 \times 3}{100} = 45$, which is Sarah's interest.

Answer to Question 15

a. **Bea** is correct.
b. Ali subtracted 6 instead of adding. Cal added 6 but forgot to divide the 6 by 4. Dev multiplied by 4 instead of dividing.
c. $y = 4 \times 3 - 6 = 6$, then $x = \frac{6+6}{4} = 3$. It works.
d. $x = \frac{22+6}{4} = \frac{28}{4} = \mathbf{7}$

The correct order is: add 6 to both sides first, then divide everything by 4.

Answer to Question 16

a. $A = \frac{1}{2}(6+10) \times 5 = \frac{1}{2} \times 16 \times 5 =$ **40 cm²**
b. $2A = (a+b)h$, so $h = \frac{2A}{a+b}$
c. $h = \frac{108}{18} =$ **6 cm**
d. $a + b = \frac{2A}{h}$, so $a = \frac{2A}{h} - b$
e. $a = \frac{80}{5} - 9 = 16 - 9 = \mathbf{7}$

To make h the subject, first multiply both sides by 2 to remove the half, then divide by the bracket $(a+b)$. To make a the subject, divide by h and then subtract b.

Check (e): $\frac{1}{2}(7+9) \times 5 = \frac{1}{2} \times 16 \times 5 = 40$, as required.

Answer to Question 17

a. Ali: $C = 40 + 30h$. Beth: $C = 70 + 20h$.
b. Ali: $40 + 60 =$ **£100**. Beth: $70 + 40 =$ **£110**.
c. $40 + 30h = 70 + 20h \Rightarrow 10h = 30 \Rightarrow h =$ **3 hours** (both £130).
d. For more than 3 hours **Beth** is cheaper, because her hourly rate is lower.
e. $h = \frac{C - 70}{20} = \frac{150 - 70}{20} =$ **4 hours**

The cheaper choice depends on the length of the job: Ali is cheaper below 3 hours

because his callout fee is lower, Beth is cheaper above 3 hours because her hourly rate is lower.

Answer to Question 18

a. Distance on a full tank = 10 × miles per gallon: Fiesta **500 miles**, Estate **350 miles**, Van **280 miles**.
b. Divide both sides of $d = Gm$ by m: $G = \frac{d}{m}$.
c. $G = \frac{245}{35}$ =**7 gallons**, leaving 3 gallons in the tank.
d. $d = Gm = 6.5 \times 28$ =**182 miles**.

Answer to Question 19

a. Plan A: $C = 25m$. Plan B: $C = 60 + 15m$.
b. A: $25 \times 6 =$ **£150**. B: $60 + 90 =$ **£150**.
c. $25m = 60 + 15m \Rightarrow 10m = 60 \Rightarrow m =$ **6 months**
d. A: $25 \times 12 = £300$. B: $60 + 180 = £240$. **Plan B** is cheaper by **£60**.

Before 6 months Plan A is cheaper; after 6 months Plan B is cheaper, because its monthly cost is £10 lower.

Answer to Question 20

a. $S = 2 \times 9 + 4 \times 3 \times 5 = 18 + 60 =$ **78**
b. $S - 2x^2 = 4xh$, so $h = \frac{S - 2x^2}{4x}$
c. $h = \frac{150 - 50}{20} = \frac{100}{20} =$ **5**
d. $h = \frac{240 - 72}{24} = \frac{168}{24} =$ **7 cm**
e. The letter x appears twice, once squared and once as a plain term, so it cannot be isolated with the operations met in Year 8 (it needs a quadratic equation).

Check (d): $2 \times 36 + 4 \times 6 \times 7 = 72 + 168 = 240$ cm², the given surface area.