Test answers - Constructions and congruence

Year: Year 8 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. In any triangle the two shorter sides must add up to more than the longest side. Here $5 + 7 = 12$ (1 mark), and $12 < 13$, so the two short sides cannot meet: **no**, the triangle cannot exist (1 mark).
b. The two shorter sides give $6 + 8 = 14$, and $14 > 10$, so **yes**, the triangle can be drawn. (1 mark)
c. Test Pythagoras with the longest side as the hypotenuse: $6^2 + 8^2 = 36 + 64 = 100$ and $10^2 = 100$. The two values are equal, so **yes**, the triangle has a right angle, opposite the 10 cm side. (1 mark)

Triangle 6, 8, 10 is the 3, 4, 5 triangle doubled, a well-known right-angled triangle.

Answer to Question 2 (4 marks)

a. Draw AB 9 cm long. Open the compasses to more than half of AB, so more than 4.5 cm. With the point on A, draw arcs above and below the line; keeping the same radius, repeat from B (1 mark for correct arcs). Join the two points where the arcs cross with a straight line (1 mark). This line cuts AB at 4.5 cm and meets it at 90°.
b. Draw the longest side, 8 cm, as the base. Set the compasses to 6 cm and draw an arc from one end; set them to 7 cm and draw an arc from the other end (1 mark for the base and both arcs). Join the crossing point to both ends of the base (1 mark).

Leave all construction arcs visible; erasing them loses marks.

Answer to Question 3 (4 marks)

a. Congruent triangles have matching sides equal. PQ matches AB, because P matches A and Q matches B, so $PQ =$**9 cm**. (1 mark)
b. AC matches PR in the same way, so $AC =$**12 cm**. (1 mark)
c. Matching angles are equal, and A matches P, so angle $A = 48^\circ$. (1 mark)
d. The test is **SAS** (side, angle, side). The known angle must be the included angle, the one between the two known sides. (1 mark)

The order of the letters is important: writing ABC ≅ PQR tells you which vertices correspond. If the angle is not between the two sides, the information does not always fix one triangle, so it is not a valid test.

Answer to Question 4 (4 marks)

a. Every point on the bisector of an angle is the same distance from both arms of the angle. The bench must be on the **angle bisector** of the two paths. (1 mark)
b. Put the compass point on O and draw an arc that crosses both paths. From each of these two crossing points draw arcs of equal radius that meet between the paths (1 mark for the arcs). Draw a straight line from O through the point where these arcs meet (1 mark).
c. The bisector cuts the 80° angle into two equal parts: $80 \div 2 =$**40°** with each path. (1 mark)

Check with a protractor: both angles should measure 40°, within a couple of degrees.

Answer to Question 5 (4 marks)

a. The seam goes to the midpoint of the base, so it splits the 6 m base into two parts of 3 m. Each small triangle has a sloping side of 5 m, a base of 3 m and the seam as its third side (1 mark). All three pairs of sides are equal, so the triangles are congruent by **SSS** (1 mark).
b. Each half is a right-angled triangle with hypotenuse 5 m and base 3 m: seam $= \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} =$**4 m**. (1 mark)
c. The seam is the perpendicular height of the whole sail: area $= \frac{1}{2} \times 6 \times 4 =$**12 m²**. (1 mark)

Each half has area 6 m², and $6 + 6 = 12$ confirms the answer.