Worksheet - Constructions and congruence

Year: Year 8 · 20

Show your working.

Question 1

Question 1 – Does the triangle exist? *(Section A · Skills practice)*

Alex is trying to construct triangles with a ruler and compasses. For a triangle to exist, the two shorter sides must add up to more than the longest side. Decide whether a triangle can be made with these side lengths. Show the addition for each.

a. 4 cm, 5 cm, 6 cm
b. 3 cm, 4 cm, 8 cm
c. 5 cm, 5 cm, 10 cm
d. 7 cm, 8 cm, 12 cm
e. 2.5 cm, 3 cm, 4.5 cm

Question 2

Question 2 – Constructing a triangle with SSS *(Section A · Skills practice)*

Construct a triangle PQR with $PQ = 7$ cm, $QR = 5$ cm and $PR = 6$ cm. Follow these instructions and answer the questions.

a. Draw the line PQ 7 cm long with a ruler. What should you do with your compasses next to draw the point R?
b. Which two arcs should you draw, and where do they cross?
c. Measure angle Q in your triangle. It should be about 57°. Explain why you must keep your construction arcs.
d. Check that your triangle is possible by adding the two shorter sides.

Question 3

Question 3 – Bisecting lines and angles *(Section A · Skills practice)*

Answer these questions about the two standard constructions. Draw each one in your book first.

a. Draw a line AB of length 8 cm and construct its perpendicular bisector. How far from A is the midpoint?
b. Draw an angle of 70° with a protractor and bisect it with compasses. What is the size of each half?
c. Draw an angle of 90° using a perpendicular, and bisect it. What angle do you get?
d. Why are the arcs in the perpendicular bisector construction always drawn with

the same radius for both ends?

Question 4

Question 4 – Spot the congruent triangles *(Section A · Skills practice)*

Two triangles are congruent if one can be moved on top of the other exactly. State which congruence test (SSS, SAS, ASA or RHS) applies to each pair, or write "not enough information".

a. Triangle 1 has sides 5 cm, 6 cm, 7 cm. Triangle 2 has sides 7 cm, 5 cm, 6 cm.
b. Triangle 1 has sides 4 cm and 6 cm with an angle of 50° between them. Triangle 2 has sides 6 cm and 4 cm with an angle of 50° between them.
c. Both triangles have angles of 40°, 60° and 80°.
d. Triangle 1 has a right angle, a hypotenuse of 13 cm and one other side of 5 cm. Triangle 2 has the same.
e. Triangle 1 has angles 40° and 70° with a side of 6 cm between them. Triangle 2 has the same.

Question 5

Question 5 – Finding missing lengths and angles *(Section A · Skills practice)*

Triangles ABC and DEF are congruent, with A matching D, B matching E and C matching F. In triangle ABC: $AB = 6$ cm, $BC = 8$ cm, $AC = 10$ cm, angle $A = 53^\circ$ and angle $B = 90^\circ$.

a. Write down the length of DE.
b. Write down the length of EF.
c. Write down the length of DF.
d. Write down angle D and angle E.
e. Work out angle C and write down angle F.

Question 6

Question 6 – Isosceles triangles *(Section A · Skills practice)*

An isosceles triangle has two equal sides and the base angles opposite them are equal. Find the angles marked with letters.

a. Apex angle 40°. Find each base angle.
b. One base angle is 65°. Find the apex angle.
c. Apex angle 100°. Find each base angle.
d. Base angles are each x and the apex angle is $3x$. Find x.

Question 7

Question 7 – The treasure hunt *(Section B · Real-life practice)*

Year 8 are organising a treasure hunt in the school grounds. The treasure is buried at a point that is exactly the same distance from the oak tree (O) and the flagpole (F), which are 12 m apart. It is also 10 m from the oak tree.

a. Draw the line segment OF using a scale of 1 cm to 2 m. How long is it on your drawing?
b. Construct the perpendicular bisector of OF. Explain why the treasure must lie on this line.
c. Draw a circle of radius 10 m (5 cm on your drawing) centred on O. Mark where it crosses the bisector.
d. There are two possible places. How far is each from the line OF, using Pythagoras' theorem? Give your answer to 1 decimal place.

Question 8

Question 8 – Placing the Wi-Fi router *(Section B · Real-life practice)*

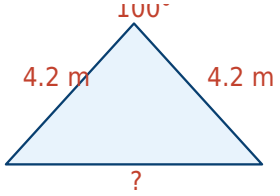
Two houses, Rose Cottage and Ivy House, are 30 m apart in a village. The parish council wants to put a Wi-Fi router at a point that is the same distance from both houses, and no more than 25 m from each.

a. Make a scale drawing with 1 cm for every 5 m. How long is the line between the houses?
b. Construct the perpendicular bisector. Why is every point on it equally far from both houses?
c. Draw arcs of radius 5 cm (25 m) from each house. How far along the bisector can the router be placed on each side of the line between the houses?
d. The router can be placed up to how many metres from the midpoint, using Pythagoras? Give your answer to 1 decimal place.

Question 9

Question 9 – The roof truss *(Section B · Real-life practice)*

A carpenter is building a wooden roof truss shaped like an isosceles triangle. The two sloping beams are each 4.2 m long and the angle at the top (the apex) is 100°.

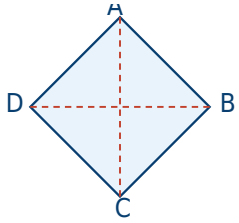


a. Find each base angle.
b. The carpenter makes a second truss to the same plan. Are the two trusses congruent? Name the test.
c. He then makes a truss with the same two beams of 4.2 m but an apex angle of 90°. Is this triangle congruent to the first one? Explain.
d. Find the base angles of the second truss.

Question 10

Question 10 – Kite making *(Section B · Real-life practice)*

Priya is making a kite from two congruent triangles. Triangle ABD and triangle CBD share the diagonal BD, with $AB = CB = 40$ cm and $AD = CD = 60$ cm.

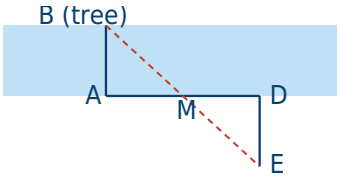


a. Explain why the two triangles are congruent. Name the test.
b. Which angles are equal as a result?
c. Angle $BAD = 110°$. Write down angle BCD.
d. Angle $ABD = 35°$. Write down angle CBD and then angle ABC.

Question 11

Question 11 – Measuring across a river *(Section B · Real-life practice)*

Fatima's class cannot walk across a river to measure its width. A tree stands at B on the far bank. They stand at A on the near bank, directly opposite the tree, and make this plan. They walk along the bank to D, 60 m from A, and mark the midpoint M of AD. At D they turn through $90°$ and walk away from the river until the tree B, the marker M and their position E are in a straight line.



a. Explain why triangles ABM and DEM are congruent.
b. Name the congruence test.
c. Which length is equal to the width AB of the river?
d. They measure $DE = 45$ m. Find the width of the river.
e. A bridge costs £800 per metre of its length. Find the cost of a bridge across the river.

Question 12

Question 12 – Shelf brackets *(Section B · Real-life practice)*

A DIY shop sells right-angled shelf brackets. Bracket A has a hypotenuse of 25 cm and one side of 7 cm. Bracket B also has a right angle, a hypotenuse of 25 cm and a side of 7 cm.

a. Which congruence test shows that the brackets are congruent?

b. Use Pythagoras' theorem to find the third side of Bracket A.
c. What is the third side of Bracket B? Explain.
d. A third bracket C has a right angle, a hypotenuse of 25 cm and a side of 15 cm. Is it congruent to A? Explain.

Question 13

Question 13 – Garden gate frames *(Section B · Real-life practice)*

A carpenter makes a gate frame as a rectangle with a diagonal brace. The rectangle is 90 cm wide and 120 cm high.

a. The brace divides the rectangle into two triangles. Prove that they are congruent.
b. Use Pythagoras to find the length of the brace.
c. The brace costs £4 per metre, and the carpenter makes 6 gates. How much does he spend on braces?
d. Why does the diagonal brace make the gate stronger than a frame without it?

Question 14

Question 14 – Ladders against the wall *(Section B · Real-life practice)*

A decorator has two identical ladders, each 5 m long. He leans each one against a vertical wall, with the foot of each ladder 3 m from the wall on level ground.

a. Use Pythagoras' theorem to find how high up the wall each ladder reaches.
b. Explain why the two triangles formed by the ladders, the wall and the ground are congruent. Name the test.
c. A third ladder, also 5 m long, has its foot 4 m from the wall. How high does it reach?
d. Is the third triangle congruent to the first? Explain.

Question 15

Question 15 – Prove it: the kite diagonal *(Section C · Problem solving)*

In kite $ABCD$, $AB = CB$ and $AD = CD$. The diagonal BD is drawn. Complete this proof that the diagonal bisects the angles at B and D.

a. Write the three pairs of equal sides in triangles ABD and CBD.
b. State the congruence test.
c. Write down two pairs of equal angles that result.
d. Angle $ABC = 84^\circ$. Find angle ABD.
e. Angle $ADB = 40^\circ$. Find angle BAD.

Question 16

Question 16 – Triangle puzzle table *(Section C · Problem solving)*

Four pupils each describe a triangle. The table shows the facts they were given.

Pupil	Facts given	Is the triangle fixed?
Ali	Sides 5, 6, 7	?
Bea	Angles 50°, 60°, 70°	?
Cal	Sides 6, 8 and the 90° angle between them	?
Dev	Sides 4, 5 and a 30° angle not between them	?

a. For each pupil decide whether there is exactly one possible triangle, using a congruence test where possible.
b. Dev's information is the SSA case. Explain why it can give two different triangles.
c. Find the missing side in Cal's triangle.
d. Cal's triangle is drawn at the same scale on grid paper. Find its area.

Question 17

Question 17 – Equilateral triangle construction *(Section C · Problem solving)*

Zoe wants to construct an equilateral triangle ABC with side 7 cm, and then use it to construct angles of 60°, 30° and 120°.

a. Describe how to construct the triangle using only a ruler and compasses.
b. What is the size of each angle? Prove it using the angle sum.
c. Bisect angle A. What angle do you obtain?
d. Extend side AB past B and find the angle between BC and the extension.
e. Zoe says the same method with side 6 cm gives a triangle congruent to the first. Is she right?

Question 18

Question 18 – Bridge design *(Section C · Problem solving)*

An engineer designs a footbridge with triangular supports. Each support triangle has sides 3.0 m, 4.0 m and 5.0 m.

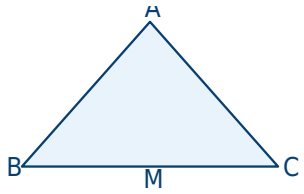
Support	Side 1	Side 2	Side 3
A	3.0 m	4.0 m	5.0 m
B	4.0 m	5.0 m	3.0 m
C	3.0 m	4.0 m	5.5 m

a. Which supports are congruent? Name the test.
b. Show that the triangle with sides 3, 4 and 5 has a right angle.
c. Support C has sides 3.0 m, 4.0 m and 5.5 m. Does it exist and is it congruent to A?
d. The bridge needs 8 identical supports like A. Find the total length of steel tube needed.

Question 19

Question 19 – Base angles proof *(Section C · Problem solving)*

In triangle ABC the sides AB and AC are equal. M is the midpoint of the base BC, and AM is drawn.



a. Write down three pairs of equal sides in triangles ABM and ACM.
b. State the test and write down which angles are equal.
c. Hence explain why the base angles of an isosceles triangle are equal.
d. Angle $ABC = 64^\circ$. Find angle BAC.

Question 20

Question 20 – Challenge: the rhombus *(Section D · Challenge)*

A rhombus $ABCD$ has all four sides equal to 10 cm. Its diagonals AC and BD cross at M. The diagonal AC is 12 cm long.

a. Prove that triangles ABD and CBD are congruent.
b. Prove that triangles ABM and CBM are congruent.
c. Hence explain why the diagonals cross at 90° and why M is the midpoint of AC.
d. Use Pythagoras to find the length of BD.
e. Find the area of the rhombus.