# Answers - Constructions and congruence

Year: Year 8

## Answer to Question 1

a. $4 + 5 = 9 > 6$: **yes**
b. $3 + 4 = 7$: **no**, the two short sides cannot reach each other
c. $5 + 5 = 10$, which equals 10: **no**, the triangle would be a flat straight line
d. $7 + 8 = 15 > 12$: **yes**
e. $2.5 + 3 = 5.5 > 4.5$: **yes**

## Answer to Question 2

a. Set the compasses to 6 cm (the length $PR$). Put the point on $P$ and draw an arc above the line.
b. Draw a second arc of radius 5 cm centred on $Q$. Where the two arcs cross is the point $R$. Join $PR$ and $QR$.
c. The arcs are the evidence of your method, so the examiner can see that you used compasses, not a protractor. The angle at $Q$:
$\cos Q = (7^2 + 5^2 - 6^2)/(2 \times 7 \times 5) = 38/70$, so $Q \approx 57.1^\circ$.
d. $5 + 6 = 11 > 7$, so the triangle exists.

## Answer to Question 3

a. The midpoint is **4 cm** from $A$. (Open the compasses to more than 4 cm, draw arcs from both ends above and below, and join the two crossing points.)
b. $70 \div 2 = 35^\circ$ each. (Draw an arc across both arms, then arcs of the same radius from each crossing point, and join the vertex to where those arcs meet.)
c. $90 \div 2 = 45^\circ$
d. Using the same radius makes every point on the arcs the same distance from $A$ as from $B$. The crossing points are therefore equidistant from both ends, and the line through them is the set of all such points.

## Answer to Question 4

a. Three pairs of equal sides: **SSS**
b. Two sides and the included angle: **SAS**
c. Only angles, no side: **not enough information** (AAA gives similar triangles, not congruent)
d. Right angle, hypotenuse and one side: **RHS**
e. Two angles and the side between them: **ASA**

## Answer to Question 5

a. $DE = AB =$ **6 cm**
b. $EF = BC =$ **8 cm**
c. $DF = AC =$ **10 cm**
d. Angle $D = A = 53^\circ$ and angle $E = B = 90^\circ$
e. Angle $C = 180 - 90 - 53 = 37^\circ$, so angle $F = 37^\circ$

## Answer to Question 6

a. $(180 - 40) \div 2 = 70^\circ$
b. $180 - 2 \times 65 = 50^\circ$
c. $(180 - 100) \div 2 = 40^\circ$
d. $x + x + 3x = 180 \Rightarrow 5x = 180 \Rightarrow x = 36^\circ$

## Answer to Question 7

a. $12 \div 2 =$ **6 cm** on the drawing.
b. Every point on the perpendicular bisector is the same distance from both ends, so the treasure (equidistant from O and F) lies on it.
c. The circle crosses the bisector in two places, one above and one below the line $OF$.
d. The midpoint $M$ is 6 m from $O$, so by Pythagoras $d^2 = 10^2 - 6^2 = 64$ and $d =$ **8 m** (exactly). Both places are 8 m from the line $OF$.

## Answer to Question 8

a. $30 \div 5 =$ **6 cm**
b. The bisector is the set of points equidistant from both ends, so any router placed on it is the same distance from both houses.
c. The arcs cross the bisector at two points; the router must lie between them.
d. Half the distance is 15 m. $d^2 = 25^2 - 15^2 = 400$, so $d =$ **20 m** from the midpoint (in each direction).

## Answer to Question 9

a. $(180 - 100) \div 2 = 40^\circ$
b. Yes: same two sides and the same included angle, so **SAS**.
c. No. The included angle is different ($90^\circ$ against $100^\circ$), so the third side is shorter and the triangles are not congruent.
d. $(180 - 90) \div 2 = 45^\circ$

## Answer to Question 10

a. Three pairs of equal sides: $AB = CB$, $AD = CD$ and $BD$ is common. This is **SSS**.
b. Angle $BAD$ = angle $BCD$, angle $ABD$ = angle $CBD$, and angle $ADB$ = angle $CDB$.
c. Angle $BCD = 110^\circ$
d. Angle $CBD = 35^\circ$, so angle $ABC = 35 + 35 = 70^\circ$

## Answer to Question 11

a. Angle $A$ = angle $D = 90^\circ$; $AM = MD$ because $M$ is the midpoint; and angle $AMB$ = angle $DME$ because they are vertically opposite.
b. **ASA**: two angles and a corresponding side.
c. Congruent triangles have equal corresponding sides, so $AB = DE$.
d. $AB = DE =$ **45 m**
e. $45 \times 800 =$ **£36 000**

## Answer to Question 12

a. **RHS**: right angle, hypotenuse and one other side are equal.
b. $\sqrt{25^2 - 7^2} = \sqrt{625 - 49} = \sqrt{576} =$ **24 cm**
c. **24 cm** too, because congruent triangles have all three sides equal.
d. No. The side of 15 cm is different from 7 cm, so the third side is $\sqrt{625 - 225} = 20$ cm and the bracket has a different shape.

## Answer to Question 13

a. Both triangles have the same width, the same height and a right angle between them (SAS), and the brace is a shared side, so they are congruent. A rectangle's opposite sides are also equal (SSS).
b. $\sqrt{90^2 + 120^2} = \sqrt{8100 + 14400} = \sqrt{22500} =$ **150 cm**
c. $6 \times 1.5 = 9$ m, so $9 \times 4 =$ **£36**
d. A triangle cannot change shape without changing the length of a side, so the two triangles hold the gate rigid, unlike a rectangle which can lean.

## Answer to Question 14

a. $\sqrt{5^2 - 3^2} = \sqrt{16} =$ **4 m**
b. Each triangle has a right angle, the same hypotenuse (5 m) and the same side on the ground (3 m), so they are congruent by **RHS**.

c. $\sqrt{5^2 - 4^2} = \sqrt{9} =$ **3 m**
d. Yes. Its sides are 3 m, 4 m and 5 m, the same three lengths as the first triangle, so they are congruent by **SSS**. The ladder is simply leaning at a different angle: it is steeper in the first triangle, because it reaches 4 m, and flatter in the third, because it reaches only 3 m.

## Answer to Question 15

a. $AB = CB$ (given), $AD = CD$ (given), $BD = BD$ (common side).
b. **SSS**
c. Angle $ABD$ = angle $CBD$ and angle $ADB$ = angle $CDB$ (corresponding angles of congruent triangles).
d. $84 \div 2 = 42^\circ$
e. In triangle $ABD$: angle $BAD = 180 - 42 - 40 = 98^\circ$

## Answer to Question 16

a. Ali: **yes** (SSS). Bea: **no**, AAA allows many sizes. Cal: **yes** (SAS). Dev: **not always**, the angle is not between the two sides.
b. The arc of radius 4 can cross the other side in two places, giving a long and a short triangle, so SSA does not guarantee congruence (unless the angle is $90^\circ$, which is RHS).
c. $\sqrt{6^2 + 8^2} = \sqrt{100} =$ **10**
d. $\frac{1}{2} \times 6 \times 8 =$ **24 square units**

## Answer to Question 17

a. Draw $AB = 7$ cm. With the compasses open to 7 cm, draw an arc from $A$ and an arc from $B$; they cross at $C$. Join $CA$ and $CB$.
b. All three sides are equal, so all three angles are equal. $180 \div 3 = 60^\circ$.
c. $60 \div 2 = 30^\circ$
d. Angle on a straight line: $180 - 60 = 120^\circ$
e. **No.** The 6 cm triangle has shorter sides, so it is similar but not congruent. Congruent triangles must have exactly the same side lengths.

## Answer to Question 18

a. A and B are congruent (**SSS**), as they have the same three side lengths in a different order.
b. $3^2 + 4^2 = 9 + 16 = 25 = 5^2$, so by the converse of Pythagoras' theorem the triangle has a right angle.

c. $3 + 4 = 7 > 5.5$, so it exists. It is **not** congruent to A because 5.5 is not 5.
d. Perimeter of A $= 12$ m, so $8 \times 12 =$ **96 m**

## Answer to Question 19

a. $AB = AC$ (given), $BM = MC$ ($M$ is the midpoint) and $AM$ is common.
b. **SSS**; angle $ABM =$ angle $ACM$.
c. Angle $ABM$ is the base angle at $B$ and angle $ACM$ is the base angle at $C$; congruent triangles have equal corresponding angles, so the base angles are equal.
d. $180 - 2 \times 64 = 52^\circ$

## Answer to Question 20

a. $AB = CB$, $AD = CD$ (all sides are 10 cm) and $BD$ is a common side, so the triangles are congruent by **SSS**. Hence angle $ABD =$ angle $CBD$.
b. $AB = CB$, angle $ABM =$ angle $CBM$ (from part a) and $BM$ is common, so the triangles are congruent by **SAS**.
c. Corresponding sides are equal, so $AM = CM = 6$ cm and $M$ is the midpoint. Angle $AMB =$ angle $CMB$ and they lie on a straight line, so each is $180 \div 2 = 90^\circ$.
d. $BM = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$, so $BD = 2 \times 8 =$ **16 cm**
e. Area $= \frac{1}{2} \times 12 \times 16 =$ **96 cm²**