# Lesson - Circles: circumference and area

Year: Year 8

This lesson covers the parts of a circle, circumference, area, semicircles, sectors and composite shapes. Keep the exact value $\pi$ until the last step, then round as the question asks.

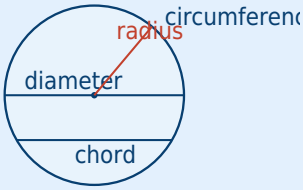
### Starter activity

Find a round object such as a tin, a mug or a plate. Measure the distance across it (the diameter) and the distance round it (the circumference) with string and a ruler. Divide the circumference by the diameter. What number do you get for each object? What do you notice?

## Parts of a circle

### Definition: circle, radius, diameter and circumference

A **circle** is the set of all points at the same distance from a fixed point, the **centre**. The **radius** $r$ is the distance from the centre to the edge. The **diameter** $d$ goes across the circle through the centre, so $d = 2r$. The **circumference** is the distance around the edge.




### Definition: chord, arc, sector and semicircle

A **chord** joins two points on the edge, an **arc** is part of the circumference, a **sector** is a slice like a piece of pizza and a **semicircle** is half a circle.

### Definition: pi

For every circle, the circumference divided by the diameter is the same number, called $\pi$ (pi). It is an irrational number that starts 3.14159... and never ends or repeats. Use the $\pi$ button on your calculator, or write answers "in terms of $\pi$" to keep them exact.

## Circumference

**Rule: circumference of a circle**

$$C = \pi d \qquad \text{or} \qquad C = 2\pi r$$

**Worked example 1: circumference**

**Find the circumference of a circle with radius 7 cm.**

$C = 2 \times \pi \times 7 = 14\pi = 44.0$ cm to 1 decimal place. In terms of $\pi$, the exact answer is $14\pi$ cm. If the diameter is 10 cm instead, $C = 10\pi = 31.4$ cm (1 d.p.).

## Area of a circle

**Rule: area of a circle**

$$A = \pi r^2$$

Square the radius first, then multiply by $\pi$.

**Worked example 2: area from the diameter**

**A circle has diameter 12 cm. Find its area.**

First halve the diameter: $r = 6$. Then $A = \pi \times 6^2 = 36\pi = 113.1$ cm$^2$ (1 d.p.).

**Watch out: radius and diameter**

A very common mistake is to square $\pi r$ or to put the diameter into the area formula. Always check whether you were given the radius or the diameter.

## Working backwards

**Method: finding the radius or diameter**

1. Write the formula that links the given measurement to $r$ or $d$.
2. Rearrange: $d = \dfrac{C}{\pi}$ or $r = \sqrt{\dfrac{A}{\pi}}$.
3. Substitute and calculate, then round at the end.

**Worked example 3: from area**

**A circle has area 154 cm$^2$. Find its radius to 1 decimal place.**

$r^2 = \dfrac{154}{\pi} \approx 49.0$, so $r = \sqrt{49.0} \approx 7.0$ cm. If instead $C = 18\pi$, then $2\pi r = 18\pi$, so $r = 9$ and $A = 81\pi$.

## Semicircles and quarter circles

### Key fact: fractions of a circle

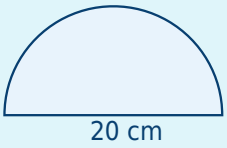
Use the fraction of the circle. A semicircle is $\dfrac{1}{2}$ of a circle and a quarter circle is $\dfrac{1}{4}$. The perimeter also includes the straight edges, so do not forget them.

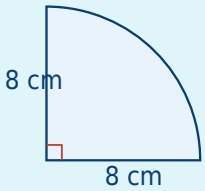
### Worked example 4: semicircle and quarter circle

**Find the area and perimeter of a semicircle of radius 10 cm, and of a quarter circle of radius 8 cm.**

Semicircle area: $\dfrac{1}{2}\pi \times 10^2 = 50\pi = 157.1$ cm$^2$. Semicircle perimeter: $\dfrac{1}{2} \times 2\pi \times 10 + 20 = 10\pi + 20 = 51.4$ cm.

Quarter circle area: $\dfrac{1}{4}\pi \times 64 = 16\pi = 50.3$ cm$^2$. Perimeter: $\dfrac{1}{4} \times 2\pi \times 8 + 8 + 8 = 4\pi + 16 = 28.6$ cm.




## Sectors

### Rule: arc length and sector area

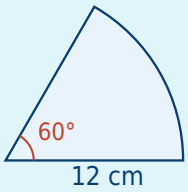
A sector with angle $\theta$ is the fraction $\dfrac{\theta}{360}$ of a full circle.

Arc length $= \dfrac{\theta}{360} \times 2\pi r$ and sector area $= \dfrac{\theta}{360} \times \pi r^2$.

### Worked example 5: a sector

**Find the arc length and area of a sector with angle $60^\circ$ and radius 12 cm.**

The fraction is $\dfrac{60}{360} = \dfrac{1}{6}$. Arc length $= \dfrac{1}{6} \times 24\pi = 4\pi = 12.6$ cm. Area $= \dfrac{1}{6} \times 144\pi = 24\pi = 75.4$ cm$^2$.




## Composite shapes

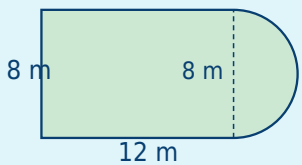
### Method: composite shapes

1. Split the shape into rectangles, triangles and parts of circles.
2. Find each area (or length).
3. Add the parts, or subtract a part that has been cut out.

### Worked example 6: a lawn

**A lawn is a rectangle 12 m by 8 m with a semicircle of diameter 8 m added to one short end. Find its area.**

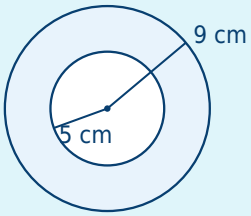
Rectangle: $12 \times 8 = 96$ m$^2$. Semicircle: radius 4, so $\dfrac{1}{2}\pi \times 16 = 8\pi = 25.1$ m$^2$. Total $= 121.1$ m$^2$.




### Worked example 7: a ring

**A ring has outer radius 9 cm and inner radius 5 cm. Find its area.**

Big circle minus small circle: $81\pi - 25\pi = 56\pi = 175.9$ cm$^2$.




## Worked example 8: wheel turns

**A wheel has diameter 70 cm. How far does it travel in 10 turns? How many turns does a 60 cm wheel need to travel 1 km?**

One turn is $70\pi \approx 219.9$ cm, so 10 turns is about 2199 cm, roughly 22 m. For the 60 cm wheel, one turn is $60\pi \approx 188.5$ cm, and $100\,000 \div 188.5 \approx 530.5$, so 531 turns are needed to go at least 1 km.

## Note: rounding

Keep the full value on your calculator until the final step. Give your answer to the accuracy requested: 1 decimal place, 2 decimal places or in terms of $\pi$. For whole items, such as turns of a wheel, round up when the answer must be at least a certain amount.

## Watch out: common mistakes

- Using the diameter in the area formula.
- Mixing up circumference and area formulae.
- Forgetting units: cm for length and $\text{cm}^2$ for area.
- Leaving out the straight sides when finding the perimeter of a semicircle.
- Rounding $\pi$ to 3 too early, which loses accuracy.

## Worked example 9: the rolling wheel

**A wheel has a radius of 35 cm. How many complete turns does it make in 500 m?**

The circumference is $2\pi \times 35 = 70\pi = 219.9$ cm $= 2.199$ m. Number of turns $= 500 \div 2.199 = 227.4$. So the wheel makes **227** complete turns (the rounding is down, because only complete turns count).

## Worked example 10: a circle drawn in terms of pi

**A circle has circumference $18\pi$ cm. Find its area in terms of $\pi$.**

$C = 2\pi r$, so $2\pi r = 18\pi$ and $r = 9$ cm. Then $A = \pi r^2 = \pi \times 81 = 81\pi$ cm². Notice that $\pi$ never has to be turned into a decimal, which keeps the answer exact.

**Worked example 11: a shape with a hole**

**A metal washer is a circle of radius 2.5 cm with a hole of radius 1 cm cut from the middle. Find the area of metal.**

Large circle: $\pi \times 2.5^2 = 6.25\pi$. Hole: $\pi \times 1^2 = \pi$. Metal $= 6.25\pi - \pi = 5.25\pi = 16.5$ cm² to 1 decimal place. You can also factorise: $\pi(2.5^2 - 1^2) = \pi \times 5.25$.

**Note: which formula do I need?**

Ask yourself: am I measuring a distance *around* the edge (a length in cm, m or km) or the amount of space *inside* (an area in cm², m² or km²)? Use $C = \pi d$ or $2\pi r$ for a length, and $A = \pi r^2$ for an area. If a question gives a diameter, halve it before you use the area formula. If it asks for a perimeter of a semicircle, do not forget to add the straight edge.

**Method: a circle problem in four steps**

1. Sketch the shape and mark every length you are given. Decide whether each length is a radius or a diameter.
2. Choose the formula: circumference for a length round the edge, area for the space inside.
3. Substitute, keep the full calculator display, and round only at the end.
4. Write the units, and check that the answer is sensible: the circumference of a circle is always a little more than three times its diameter.

**Watch out: pi and rounding**

- Use the π button on your calculator, not 3 or 3.14, unless the question tells you to. Do not round until the very last step.
- The area formula is $\pi r^2$, meaning $\pi \times (r \times r)$, not $(\pi r)^2$ and not $2\pi r$.
- When a question asks for how many *complete* turns or laps, round down; when it asks for how many tins or packs to buy, round up.

**Did you know?**

The number $\pi$ is irrational, which means its decimal digits go on for ever without repeating. In 2022 computers calculated it to more than 100 trillion digits. For almost every real-life job, 10 digits are more than enough: using just 15 digits of $\pi$ you can work out the circumference of the Earth to within the width of a pinhead.

## Checking circle answers

Because π is a little more than 3, you can always check a circle calculation in your head. The circumference is just over three times the diameter, and the area is just over three

times the square of the radius. For a circle of radius 10 cm, the circumference should be a bit more than 60 cm (it is 62.8 cm) and the area a bit more than 300 cm² (it is 314.2 cm²). If your calculator gives an answer far from these estimates, you have probably used the diameter instead of the radius or forgotten to square.

**Did you know?**

The ancient Greek mathematician Archimedes estimated π more than 2 200 years ago by drawing polygons with 96 sides inside and outside a circle. He proved that π lies between $3\frac{10}{71}$ and $3\frac{1}{7}$. Today computers have calculated π to more than a hundred trillion digits, and the digits never repeat.

**Otto's tip**

Circumference is a length, so it has one "r". Area covers a surface, so it has an "r" squared. Check that your units match: cm for circumference, cm squared for area.